Rewriting Ontology-Based Mappings

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Abstract

Data translation consists of the task of moving data from a source database to a target database. This task is usually performed by developing mappings, i.e. executable transformations from the source to the target schema. However, a richer description of the target database semantics may be available in the form of an ontology. This is typically defined as a set of views over the base tables that provides a unified conceptual view of the underlying data. We investigate how the mapping process changes when such a rich conceptualization of the target database is available. We develop a translation algorithm that automatically rewrites a mapping from the source schema to the target ontology into an equivalent mapping from the source to the target databases. Then, we show how to handle this problem when an ontology is available also for the source. Differently from previous approaches, the language we use in view definitions has the full power of non-recursive Datalog with negation. In the paper, we study the implications of adopting such an expressive language. Experiments are conducted to illustrate the trade-off between expressibility of the view language and efficiency of the chase engine used to perform the data exchange.

1 Introduction

Integrating data coming from disparate sources is a crucial task in many applications. An essential requirement of any data integration task is that of manipulating *mappings* between sources. Mappings are executable transformations that define how an instance of a source repository can be translated into an instance of a target repository. Traditionally, mappings are developed to exchange data between two relational database schemas [19]. A rich body of research has been devoted to the study of this subject. This includes the development of algorithms to simplify the specification of the mapping [22], the formalization of the semantics of the translation process [8], and various notions of quality of the results [26, 16, 15].

This paper investigates how the mapping process changes in the presence of richer *ontology schemas* of the two data sources. Studying this variant of the problem is important for several reasons.

(*i*) First, the emergence of the Semantic Web has increased the number of data sources on top of which ontology-like descriptions are developed.

(*ii*) Second, ontologies play a key role in information integration since they are used to give clients a global conceptual view of the underlying data, which in turn may come from external, independent, heterogeneous, multiple information systems [14]. On the contrary, the global unified view given by the ontology is constructed independently from the representation adopted for the data stored at the sources.

(*iii*) Finally, many of the base transactional repositories used in complex organizations by the various processes and applications often undergo modifications during the years, and may lose their original design. The new schema can often be seen as a set of views over the original one. It is important to be able to run the existing mappings against a view over the new schema that does not change, thus keeping these modifications of the sources transparent to the users.

It is therefore important to study how the mapping process changes in this setting.

1.1 Contributions

In this paper, we assume that an ontology is provided for the target and, possibly, for the source data repository. The relationship between the domain concepts in this ontology schema and the data sources is given by a set of views that define the ontology constructs in terms of the logical database tables using a relational language of conjunctive queries, comparisons and negations.

We develop a number of techniques to solve this kind of *ontology-based mapping problem*. More specifically:

- we develop rewriting algorithms to automatically translate mappings over the ontology schema into mappings over the underlying databases; we first discuss the case in which an ontology schema is available for the target database only; then we extend the algorithm to the case in which an ontology schema is available both for the source and the target;
- the algorithm that rewrites a source-to-ontology mapping into a classical and executable source-to-target mapping is based on the idea of unfolding views in mapping conclusions; in our setting this unfolding is far from being straightforward; in the paper, we show that the problem is made significantly more complex by the expressibility of the view-definition language, and more precisely, by the presence of negated atoms in the body of view definitions;
- we study the implications of adopting such an expressive language; to handle negation in view definitions we adopt a very expressive mapping language, namely, that of *disjunctive embedded dependencies (deds)* [3]. Deds are mapping dependencies that may contain disjunctions in their heads, and are therefore more expressive than standard embedded dependencies (tgds and egds);
- this increased expressive power makes the data-exchange step significantly more complex. As a consequence, we investigate restrictions to the view-definition language that may be handled using standard embedded dependencies, for which efficient execution strategies exist. In the paper, we identify a restricted view language that still allows for a limited form of negation, but represents a good compromise between expressibility and complexity; we prove that under this language, our rewriting algorithm always returns standard embedded dependencies;
- the classical approach to executing a source-to-target exchange consists of running the given mappings using a *chase* engine [8]. We build on the LLUNATIC chase engine [11, 12], and extend it to execute not only standard tgds and egds, but also deds. We discuss the main technical challenges related to the implementation of deds. Then, using the prototype, we conduct several experiments on large databases and mapping scenarios to show the trade-offs between expressibility of the view language, and efficiency of the chase. To the best of our knowledge, this is the first practical effort to implement execution strategies for deds, and may pave the way for further studies on the subject.

This paper represents a significant step forward towards the goal of incorporating richer ontology schemas into the data translation process. Given the evolution of the Semantic Web, and the increased adoption of ontologies, this represents an important problem that may lead to further research directions.

This paper extends our prior research [18], where we first studied the problem of rewriting ontology-based mappings. We make several important advancements, as follows:

(*i*) First, previous papers only discussed rewritings based on standard embedded dependencies for a rather limited form on negation. In this paper, we extend our algorithms to handle arbitrary non-recursive Datalog with negation using deds, thus considerably extending the reach of our rewriting algorithm.

(*ii*) At the same time, we make the sufficient conditions under which the rewriting only contains embedded dependencies more precise, and extend the limited case discussed in previous papers.

(*iii*) In addition, we present the first chase technique for deds, and a comprehensive experimental evaluation based on scenarios with and without deds. As we mentioned above, this is the first practical study of the scalability of the chase of high-complexity dependencies, an important problem in data exchange.

(*iv*) Finally, we provide full proofs of all theorems (in A).

1.2 Outline

The paper is organized as follows. Our motivating example is given in Section 2. Section 3 recalls some basic notions and definitions. Section 4 introduces the ontology-based mapping problem. Section 5 defines disjunctive embedded

dependencies which are required by the rewriting when the views that define the mapping are beyond conjunctive queries. Section 6 provides the definition of a correct rewriting. The rewriting algorithm and formal results are in Section 7. Section 8 identifies a view-definition language that is more expressive than plain conjunctive queries but such that it computes correct rewritings only in terms of embedded dependencies. The chase engine is described in Section 9. Experiments are in Section 10. We discuss related work in Section 11.

2 Motivating Example

Assume we have the two relational schemas below and we need to translate data from the source to the target.

Source schema: *S-WorkerGrades*(*WorkerId*, *Year*, *Grade*, *SalaryInc*)
S-Stats(*WorkerId*, *WorkerName*, *MinGrade*, *MaxGrade*)

Target schema: *Employees*(*Id*, *Name*)
Evaluations(*EmployeeId*, *Year*)
PositiveEvals(*EmployeeId*, *Year*, *SalaryInc*)
Penalized(*EmployeeId*, *Year*)
Warned(*EmployeeId*, *Date*)

Both schemas rely on the same domain, which includes data about employees and the evaluations they receive during the years. The source database stores grades within the *S-WorkerGrades* table, and statistical data in the form of minimum and maximum grades of workers in table *Stats*. The target database, on the contrary, stores data about employees and their positive evaluations, but also records warnings and penalties for those employees.

Due to these different organizations, it is not evident how to define the source-to-target mapping. In particular, it is difficult to relate information stored in table *S-Stats* from the source schema to the contents of the tables *Penalized* and *Warned* in the target schema.

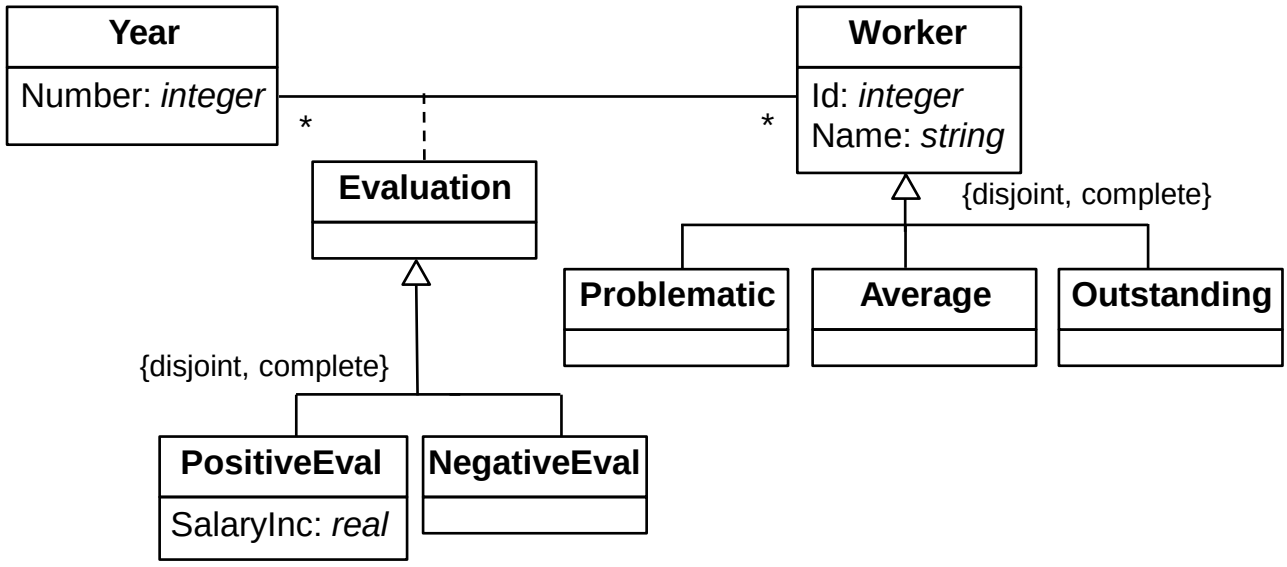


Figure 1: A Simple Target Ontology.

Suppose now that a richer ontology has been defined over the target relational schema, as shown in Figure 1. The ontology distinguishes among problematic, average, and outstanding workers, and it records whether the yearly evaluation of each worker is negative or positive, storing also the salary increase to apply to the worker for positive evaluations.

Each class and association in the ontology is defined in terms of the database tables by means of a set of views, as follows (to simplify the reading, from now on we use different fonts for ontology classes and *relational tables*; in addition, source tables have a *S-*prefix in their name to be distinguished from base target tables): [1]

[1] The rules we use to specify views in our example are not safe in the sense that they contain variables appearing in negative literals that do not appear in a positive one. This is done for the sake of readability since it is well-known that there is an equivalent safe rewriting for such rules.

View definitions for the target ontology.

v_1 : Worker(id, name) $\Leftarrow$ *Employees*(*id*, *name*)

v_2 : Evaluation(employeeId, year) $\Leftarrow$ *Evaluations*(*employeeId*, *year*)

v_3 : PositiveEval(employeeId, year, salaryInc) $\Leftarrow$ Evaluation(employeeId, year),
PositiveEvals(*employeeId*, *year*, *salaryInc*)

v_4 : NegativeEval(employeeId, year) $\Leftarrow$ Evaluation(employeeId, year),
¬PositiveEval(employeeId, year, sinc)

v_5 : Problematic(id, name) $\Leftarrow$ Worker(id, name), *Penalized*(*id*, *year*)

v_6 : Problematic(id, name) $\Leftarrow$ Worker(id, name), ¬PositiveEval(id, year, sinc)

v_7 : Outstanding(id, name) $\Leftarrow$ Worker(id, name), ¬NegativeEval(id, year),
¬ *Warned*(*id*, *date*)

v_8 : Average(id, name) $\Leftarrow$ Worker(id, name), ¬Outstanding(id, name),
¬Problematic(id, name)

The process of defining semantic abstractions over databases can bring benefits to data architects only as long as the view-definition language is expressive enough. To this end, the view-definition language adopted in this paper goes far beyond plain conjunctive queries, and has the full power of non-recursive Datalog [6] with negation. In fact:

(*i*) we allow for negated atoms in view definitions; these may either correspond to negated base tables, as happens in view v_7 (table *Warned*), or even to negated views, as in v_4 (view PositiveEval), v_6 (PositiveEval), v_7 (NegativeEval) and v_8 (Outstanding and Problematic);

(*ii*) views can be defined as unions of queries; in our example, Problematic workers are the ones that either have been penalized or have received no positive evaluations at all.

The semantics of this ontology is closer to the way the information is stored in the source schema than the one provided by the physical target tables (notice how the ontology hides tables *Penalized* and *Warned*). Therefore, the mapping designer will find it easier to define a mapping from the source schema to the target ontology. For instance, s/he could realize that the classification of workers as Average, Outstanding and Problematic in the ontology corresponds to a ranking of workers based on their grades in the source schema. In this way, employees with grades consistently above 9 (out of 10) are outstanding, those always graded less than 4 are considered to be problematic, and the rest are average.

As is common [8], we use *tuple generating dependencies (tgds)* and *equality-generating dependencies (egds)* [3] to express the mapping. In our case, the translation of source tuples into the Average, Outstanding and Problematic target concepts can be expressed by using the following tgds with comparison atoms:

m_0 : $\forall id, yr, gr, sinc, name, maxgr, mingr$:
S-WorkerGrades(*id*, *yr*, *gr*, *sinc*), *S-Stats*(*id*, *name*, *mingr*, *maxgr*),
$maxgr > 4, mingr < 9 \rightarrow$ Average(id, name)

m_1 : $\forall id, yr, gr, sinc, name, maxgr, mingr$:
S-WorkerGrades(*id*, *yr*, *gr*, *sinc*), *S-Stats*(*id*, *name*, *mingr*, *maxgr*),
$mingr \geq 9 \rightarrow$ Outstanding(id, name)

m_2 : $\forall id, yr, gr, sinc, name, maxgr, mingr$:
S-WorkerGrades(*id*, *yr*, *gr*, *sinc*), *S-Stats*(*id*, *name*, *mingr*, *maxgr*),
$maxgr \leq 4 \rightarrow$ Problematic(id, name)

m_3 : $\forall id, yr, gr, sinc$:
S-WorkerGrades(*id*, *yr*, *gr*, *sinc*), $gr \geq 5 \rightarrow$ PositiveEval(id, yr, sinc)

m_4 : $\forall id, yr, gr, sinc$:
S-WorkerGrades(*id*, *yr*, *gr*, *sinc*), $gr < 5 \rightarrow$ NegativeEval(id, yr)

Intuitively, tgd m_0 specifies that, for each pair of tuples in the source tables *S-WorkerGrades* and *S-Stats* that have the same value for the *id* attribute and have a *maxgrade* attribute greater than 4 and a *mingrade* attribute lower than 9, there should be a worker ranked as average in the ontology. Similarly for m_1 and m_2 for Outstanding and Problematic, respectively.

Mappings m_3 and m_4 relate the workers' evaluation data in *S-WorkerGrades* to the instances PositiveEval and NegativeEval, respectively, using the grade to discriminate between the two subclasses of Evaluation.

Notice that mappings m_0, m_1 and m_2 do not completely encode the semantics of the desired transformation. In fact, an important part of the mapping process is to generate *solutions*, i.e. instances of the target that comply with the integrity constraints imposed over the database. To do this, it is necessary to incorporate the specification of these constraints into the mapping itself. This can be done easily using additional dependencies. The mapping

literature [22] usually treats target dependencies in a different way. In fact, it is customary to embed foreign-key constraints into the source-to-target tgds that express the mapping. In contrast, egds require special care [15], and therefore must be expressed as separate dependencies.

Mapping e_0 below is an example of an egd used to express the key constraint on Worker: it states that whenever two workers have the same id, their names must also be the same:

$$e_0 : \forall id, name_1, name_2 : \mathsf{Worker}(\mathsf{id}, \mathsf{name}_1), \mathsf{Worker}(\mathsf{id}, \mathsf{name}_2) \rightarrow \mathsf{name}_1 = \mathsf{name}_2$$

We want to emphasize the benefits of designing the mappings wrt the richer target ontology rather than wrt to the base tables. By taking advantage of the semantics of the ontology, the mapping designer does not need to care about the physical structure of the data in the target schema. As an example, s/he does not need to explicitly state in m_0, m_1, m_2 that average, outstanding, and problematic workers are also workers, nor that a positive or negative evaluation is also an evaluation in m_3, m_4. The class-subclass relationships are encoded within the ontology schema, and we expect their semantics to carry on into the mappings.

However, this increased flexibility comes at a cost. For example, mappings m_0 to m_4 above are not directly executable, since they refer to virtual entities — the constructs in the ontology schema — and not to the actual tables in the target. We therefore need to devise a way to translate such a *source-to-ontology* mapping into a classical source-to-target mapping, in order to execute the latter and move data from the source to the target database.

The main technical problem addressed in this paper can therefore be stated as follows: given a source-to-ontology mapping, a target ontology schema, and the views defining this ontology schema in terms of the underlying database tables, we want to obtain the corresponding executable source-to-target mapping.

3 Preliminary Notions

In this paper, we deal with mapping scenarios that involve two levels: the ontology and the database level. This section first introduces the basic concepts of these two levels, and then elaborates on the language of dependencies used to express mapping scenarios.

3.1 Databases and Ontologies

Databases We focus on the relational setting. A *schema* $\mathbf{S}$ is a set of relation symbols $\{R_1, \ldots, R_n\}$, each with an associated relation schema $R(A_1, \ldots, A_m)$. Given schemas $\mathbf{S}, \mathbf{T}$ with disjoint relations symbols, $\langle \mathbf{S}, \mathbf{T} \rangle$ denotes the schema corresponding to the union of $\mathbf{S}$ and $\mathbf{T}$. An *instance* of a schema is a set of tuples in the form $R(v_1, \ldots, v_m)$, where each v_i denotes either a constant, typically denoted by $a, b, c, \ldots$, or a *labeled null*, denoted by $N_1, N_2, \ldots$. Constants and labeled nulls form two disjoint sets. Given instances I and J, a homomorphism $h : I \rightarrow J$ is a mapping from $dom(I)$ to $dom(J)$ such that for every $c \in \text{CONST}$, $h(c) = c$, and for all tuples $t = R(v_1, \ldots, v_n)$ in I, it is the case that $h(t) = R(h(v_1), \ldots, h(v_n))$ belongs to J. Homomorphisms immediately extend to formulas, since atoms in formulas can be seen as tuples whose values correspond to variables.

Ontologies In this paper, we focus on ontologies that deal with static aspects. In particular, we consider ontologies that consist of a taxonomy of entity types (which may have attributes), a taxonomy of relationship types (defined among entity types), and a set of integrity constraints (which affect the state of the domain). The integrity constraints are expressed by means of *dependencies* (see Section 3.2).

Views To bridge the gap between the ontology schema and the underlying database, we assume that a set of GAV views (Global-As-View) is given for each entity and relationship type, which defines this type in terms of the underlying database. A *view* V is a derived relation defined over a schema S. The view definition for V over S is a non-recursive rule of the form:

$$v : V(\overline{x}) \Leftarrow R_1(\overline{x}_1), \ldots, R_p(\overline{x}_p), \neg R_{p+1}(\overline{x}_{p+1}), \ldots, \neg R_{p+g}(\overline{x}_{p+g})$$

with $p \geq 1$ and $g \geq 0$, where the variables in $\overline{x}$ are taken from $\overline{x}_1, \ldots, \overline{x}_p$. Atoms in a view definition can be either base or derived. An atom $V(\overline{x})$ is a *derived atom* if V denotes a view; otherwise it is a *base atom*. A view definition specifies how the extension of the view is computed from a given instance of the underlying schema, that is, given a homomorphism h from the definition of V to an instance I, $h(V(\overline{x}))$ belongs to the extension of V iff $h(R_1(\overline{x})) \wedge \ldots \wedge \neg h(R_{p+g}(\overline{x}_{p+g}))$ is true on I.

3.2 Dependencies and Mapping Scenarios

Dependencies A *tuple-generating dependency (tgd)* over $\mathbf{S}$ is a formula of the form $\forall \overline{x}, \overline{z}\big(\phi(\overline{x}, \overline{z}) \rightarrow \exists \overline{y}\psi(\overline{x}, \overline{y})\big)$, where $\phi(\overline{x}, \overline{z})$ and $\psi(\overline{x}, \overline{y})$ are conjunctions of atoms. We allow two kinds of atoms in the premise: (a) relational atoms over $\mathbf{S}$; (b) comparison atoms of the form v *op* c, where *op* is a comparison operator ($=, >, <, \geq, \leq$), v is a variable that also appears as part of a relational atom, and c is a constant. Only relational atoms are allowed in the conclusion.

An *equality generating dependency (egd)* over $\mathbf{S}$ is a formula of the form $\forall \overline{x}(\phi(\overline{x}) \rightarrow x_i = x_j)$ where $\phi(\overline{x})$ is a conjunction of relational atoms over $\mathbf{S}$ and comparison atoms as defined above, and x_i and x_j occur in $\overline{x}$. A *denial constraint* is a special form of egd of the form $\forall \overline{x}\big(\phi(\overline{x}) \rightarrow \bot\big)$, in which the conclusion only contains the $\bot$ atom, which cannot be made true. Tgds and egds [3] form the language of *embedded dependencies*.

Mapping Scenarios A *mapping scenario* [8], $\mathcal{M} = \{\mathbf{S}, \mathbf{T}, \Sigma_{ST}, \Sigma_T\}$, is a quadruple consisting of:

- a source schema $\mathbf{S}$;
- a target schema $\mathbf{T}$;
- a set of *source-to-target (s-t) tgds* Σ_{ST}, i.e. tgds such that the premise is a formula over $\mathbf{S}$ and the conclusion a formula over $\mathbf{T}$;
- a set Σ_T of *target tgds* — tgds over $\mathbf{T}$ — and *target egds* — egds over $\mathbf{T}$.

Given a source instance I, a solution for I under $\mathcal{M}$ is a target instance J such that I and J satisfy Σ_{ST}, and J satisfies Σ_T. A solution J for I and $\mathcal{M}$ is called a *universal solution* if, for all other solutions J' for I and $\mathcal{M}$, there is a homomorphism from J to J'. The chase is a well-known algorithm for computing universal solutions [8]. We denote by $\mathsf{Sol}(\mathcal{M}, I)$ the set of solutions for $\mathcal{M}$ and I, and by $\mathsf{USol}(\mathcal{M}, I)$ the set of universal solutions for $\mathcal{M}$ and I.

4 The Ontology-Based Mapping Problem

The goal of this section is to introduce our mapping problem. Let us first assume that an ontology schema is only available for the target database (case a). Then, we discuss how things can be extended to handle a source ontology as well (case b).

4.1 Case a: Source-to-Ontology Mappings

The inputs to our source-to-ontology mapping problem are:

1. a source relational schema, $\mathbf{S}$, and a target relational schema $\mathbf{T}$;
2. a target ontology schema, $\mathbf{V}$, defined by means of a set of view definitions, Υ_{TV}, over $\mathbf{T}$. View definitions may involve negations over derived atoms, as discussed in Section 3;
3. a set of target constraints, Σ_V, i.e. target egds to encode key constraints and functional dependencies over the ontology schema;
4. finally, a source-to-ontology mapping, Σ_{SV}, defined as a set of s-t tgds over $\mathbf{S}$ and $\mathbf{V}$.

Based on these, our intention is to rewrite the dependencies in $\Sigma_{SV} \cup \Sigma_V$ as a new set of source-to-target dependencies $\Sigma_{ST} \cup \Sigma_T$, from the source to the target database. The process is illustrated in Figure 2a, where solid lines refer to inputs, and dashed lines to outputs produced by the rewriting.

4.2 Case b: Ontology-to-Ontology Mappings

The following sections are devoted to the development of the mapping rewriting algorithm. Before we turn to that, let us discuss what happens when also an ontology schema over the source is given, as shown in 2b. In this case, we assume that in addition to the target-ontology view-definitions, Υ_V, view definitions for the source ontology schema, $\Upsilon_{V'}$, are also given, with the respective egds. We also assume that the mapping, $\Sigma_{V'V}$, is designed between the two ontologies.

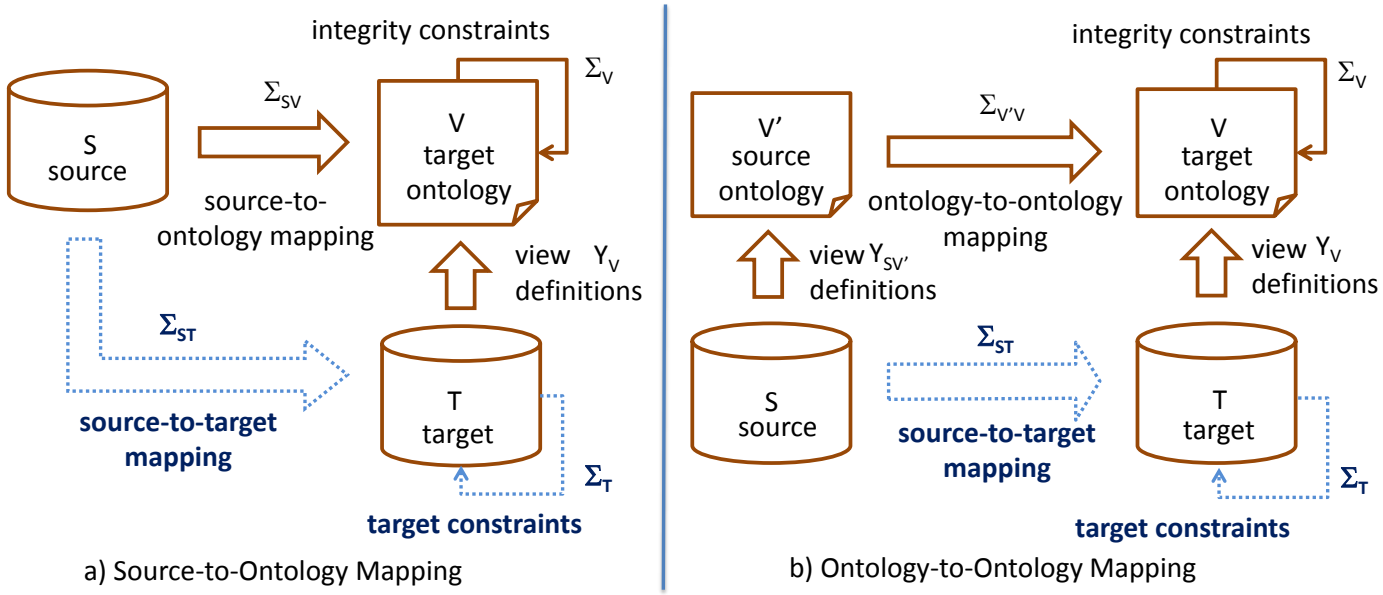


Figure 2: Ontology Mapping Scenarios.

It can be seen that this case can be reduced to the one above. We can see the problem as the composition of two steps:

(i) applying the source view definitions in $\Upsilon_{V'}$ to the source instance, I, to materialize the extent of the source ontology, $\Upsilon_{V'}(I)$;

(b) consider this materialized instance as a new source database, and solve the source-to-ontology mapping problem as in Figure 2a.

In light of this, in the following we concentrate on the scenario in Figure 2a only.

5 Disjunctive Embedded Dependencies

Mappings with views have been addressed in previous papers (e.g. [16, 26]). As is obvious, the complexity of the problem depends quite a lot on the expressibility of the view-definition language allowed in our scenarios. Previous works have made almost exclusive reference to views defined using the language of conjunctive queries. In this case, the rewriting consists of an application of the standard view unfolding algorithm [25].

To give an example, consider mapping m_3 (from now on, we omit universal quantifiers), and recall the definition of views PositiveEval, and Evaluation:

$$m_3 : \textit{S-WorkerGrades}(id, yr, gr, sinc), gr \geq 5 \rightarrow \mathsf{PositiveEval}(\mathsf{id}, \mathsf{yr}, \mathsf{sinc})$$
$$v_2 : \mathsf{Evaluation}(\mathsf{employeeId}, \mathsf{year}) \Leftarrow \textit{Evaluations}(\textit{employeeId}, \textit{year})$$
$$v_3 : \mathsf{PositiveEval}(\mathsf{employeeId}, \mathsf{year}, \mathsf{sinc}) \Leftarrow \mathsf{Evaluation}(\mathsf{employeeId}, \mathsf{year}), \textit{PositiveEvals}(\textit{employeeId}, \textit{year}, \textit{sinc})$$

Standard view unfolding replaces the view symbols of tgd conclusions by their definitions, while appropriately renaming the variables. In our example, this yields the following s-t tgd:

$$m_3' : \textit{S-WorkerGrades}(id, yr, gr, sinc), gr \geq 5 \rightarrow \textit{Evaluations}(id, yr), \textit{PositiveEvals}(id, year, sinc)$$

However, the main purpose of having a semantic description of the target database stands in its richer nature with respect to the power of the pure selection-projection-join paradigm. In this paper we allow for a more expressive language than conjunctive queries, i.e, non-recursive Datalog with negation.

It is known [4] that the language of embedded dependencies (tgds and egds) is closed wrt unfolding conjunctive views, i.e. the result of unfolding a set of conjunctive view definitions within a set of tgds and egds is still a set of tgds and egds. A natural question is if this is also true for our more expressive view-definition language. Unfortunately, we can provide a negative answer to this question.

Theorem 1 *There exists a source-to-ontology mapping scenario $\mathcal{M}_{SV} = \{\boldsymbol{S},\ \boldsymbol{V},\ \Sigma_{SV}, \Sigma_V\}$ with view definition Υ_V, and an instance I, such that $\mathcal{M}_{SV}$ and I admit a universal solution $J_V \in \mathsf{USol}(\mathcal{M}_{SV}, I)$, and there exists no source-to-target scenario $\mathcal{M}_{ST}$ composed of embedded dependencies (tgds and egds) such that $\mathcal{M}_{ST}$ and I admit a solution J_T, and $J_V = \Upsilon(J_T)$.*

The proof of the theorem is in A. Regardless of the technical details, it is quite easy to get the intuition that stands behind this negative result: in essence, we are doomed to fail in some cases because of the limited expressive power of our mapping language. In essence, we are trying to capture the semantics of a view-definition language that allows for non-recursive negation, by means of a mapping language based on embedded dependencies, that does not use negation.

This justifies two important choices wrt the algorithm:

(*i*) To start, we follow a best-effort approach. We design an algorithm that is sound, i.e. given $\mathcal{M}_{SV}$, it generates a rewritten source-to-target scenario $\mathcal{M}_{ST}$ such that, whenever $\mathcal{M}_{ST}$ admits a universal solution J_T, then also the original source-to-ontology $\mathcal{M}_{SV}$ admits universal solutions on I, and it is the case that $\Upsilon_V(J_T)$ is a solution for $\mathcal{M}_{SV}$ and I. In other terms, we give up completeness, and say nothing about the cases in which $\mathcal{M}_{ST}$ fails. This notion will be made more precise in the following.

(*ii*) To better simulate the effects of negation in view definitions, we choose a very expressive mapping language, i.e. we extend the language of embedded dependencies (tgds and egds), by introducing disjunctions in conclusions. This gives us the more expressive mapping language of *disjunctive embedded dependencies (deds)*, that we use as a target language for our rewritings, formalized as follows.

Definition 1 (Ded) *A disjunctive embedded dependency (ded) is a first-order formula of the form:*

$$\forall\, \overline{x}, \overline{z}\big(\varphi(\overline{x}, \overline{z}) \rightarrow \bigvee_{l=1}^{n} (\exists\, \overline{y_l} \psi_l(\overline{x}, \overline{y_l}))\big)$$

where $\varphi(\overline{x}, \overline{z})$ and each $\psi_l(\overline{x}, \overline{y_l})$ are conjunctions of atoms. Atoms in each conjunct $\psi_l(\overline{x}, \overline{y_l})$ may be either relational atoms, or comparison atoms of the form $(x_i = x_j)$, or the special unsatisfiable atom $\bot$.

A ded is called a source-to-target ded if $\varphi(\overline{x}, \overline{z})$ is a conjunction of relational atoms over $\boldsymbol{S}$, and each $\psi_l(\overline{x}, \overline{y_l})$ is a conjunction of relational atoms over $\boldsymbol{T}$. It is called a target ded if $\varphi(\overline{x}, \overline{z})$ is a conjunction of relational atoms over $\boldsymbol{T}$, and each $\psi_l(\overline{x}, \overline{y_l})$ is either a comparison atom, or a conjunction of relational atoms over $\boldsymbol{T}$, or the unsatisfiable atom.

In essence, the conclusion of a ded is the disjunction of various conjunctions, as in the following examples, where S_i are source symbols, and T_j are target symbols:

$$\begin{array}{l} m_{d_1} : \forall x : S_1(x) \rightarrow (\exists y : T_1(x, y)) \vee T_2(x, x) \\ m_{d_2} : \forall x, y : S_2(x, y) \rightarrow T_3(x, y) \vee (\exists z : T_3(x, z),\ T_4(z, y)) \\ m_{d_3} : \forall x, y, z, y', z' : T_1(x, y, z),\ T_1(x, y', z') \rightarrow (y = y') \vee (z = z') \\ m_{d_4} : \forall x, y, z, y', z' : T_1(x, y, z) \rightarrow (y = z) \vee T_3(x, y) \\ m_{d_5} : \forall x, y, z, y', z' : T_1(x, y, z),\ T_1(x, y', z') \rightarrow \bot \end{array}$$

Here, m_{d_1} and m_{d_2} are source-to-target deds, while m_{d_3}, m_{d_4} and m_{d_5} are target deds. The semantics is easily explained: m_{d_1} is satisfied by instances I, J of $\mathbf{S}$, $\mathbf{T}$ if, whenever there exists in I a tuple of the form $S_1(c)$, where c is a constant, then J either contains a tuple of the form $T_1(c, v)$ (where v is a constant or a labeled null), or it contains a tuple of the form $T_2(c, c)$. Similarly for m_{d_2}.

Based on this, it is easy to see that ded m_{d_3} states that table T_1 is such that, for any pair of tuples, whenever the first attributes are equal, then either the second ones, or the third ones must be equal too. In this respect, this is a generalization of an egd. It is also interesting to note that deds may freely mix equalities and relational atoms in their conclusions, as happens with m_{d_4}.

Ded m_{d_5} states what is called a *denial constraint*: since its conclusion only contains the unsatisfiable atom, then it will fail whenever the premise is satisfied, since there is no way to satisfy the constraint. It is a way to state failure conditions for the mappings, i.e. configurations of the source and target instances for which there is no solution.

Clearly the definition of deds contains, for $l = 1$, that of the classical embedded dependencies. A *mapping scenario with deds* is a quadruple $\mathcal{M}^{ded} = \{\mathbf{S}, \mathbf{T}, \Sigma_{ST}, \Sigma_T\}$ where Σ_{ST} is a set of *source-to-target deds* and Σ_T is a set of *target deds*.

There are a few important differences between ordinary mapping scenarios with embedded dependencies, and their counterpart with deds. Recall from Section 3 that the semantics of ordinary mapping scenarios is centered

around the notion of a *universal solution*. Given a scenario $\mathcal{M}^{emb}$ and a source instance I, in most cases there are countably many solutions, i.e. target instances that satisfy the dependencies. Consider for example:

$$m_1 : \forall x : S_1(x) \rightarrow \exists y : T_1(x, y)$$

Given $I = \{S_1(a)\}$, all of the following are solutions for m_1 (in the following, $a, b, c, \ldots$ are constants and N_i denotes a labeled null, i.e. a null value with an explicit label introduced to satisfy existential quantifiers):

$$J_1 = \{T_1(a, N)\} \quad J_3 = \{T_1(a, b), T_1(a, N)\}$$
$$J_2 = \{T_1(a, b)\} \quad J_4 = \{T_1(a, b), T_2(b, c)\}$$

A solution for $\mathcal{M}^{emb}$ and I is called a *universal solution* if it has a homomorphism in every other solution for $\mathcal{M}^{emb}$ and I. Universal solutions are considered as "good" solutions, preferable to non universal ones. The intuition behind the formal definition is that a universal solution does not introduce any unnecessary and unjustified information within the target. In fact, any unjustified tuples would not be mappable via homomorphisms in every other solution. In our example, only J_1 is universal; every other solution in the example contains extra information that is not strictly necessary to enforce the tgd, either in the form of constants in place of nulls, or extra tuples.

As soon as we introduce deds, the theoretical framework changes quite significantly. Deutsch and others have shown [7] that the definition of a universal solution is no longer sufficient for ded-based scenarios, and that the more appropriate notion of *universal model set* is needed.

Definition 2 (Universal Model Set) *Given an instance I under a scenario $\mathcal{M}^{ded}$, a universal model set is a set of target instances $\boldsymbol{J} = \{J_0, \ldots, J_n\}$ such that:*

- *every $J_i \in \boldsymbol{J}$ is a solution form M^{ded};*
- *for every other solution J', there exists a $J_i \in \boldsymbol{J}$ such that there is a homomorphism from J_i to J'.*

It is not difficult to understand why a set of different solutions is needed. Consider our ded m_{d_1} above. On source instance $I = \{S_1(a)\}$, it has two completely different solutions, namely $J_1 = \{T_1(a, N)\}$, $J_2 = \{T_2(a, a)\}$. Neither is universal in the ordinary sense, since they cannot be mapped into one another; on the contrary, both contribute to describe the "good" ways to satisfy m_{d_1}.

In the following, we introduce our rewriting algorithm with deds. Before turning to it, it is important to emphasize another crucial difference wrt standard embedded dependency in terms of the complexity of generating solutions. The chase [8] is a well known, polynomial-time procedure to generate universal solutions for standard tgds and egds. It is possible, as we discuss in the following sections, to extend it to generate universal model sets for deds, but at a price in terms of complexity. Universal model sets, in fact, are usually of exponential size wrt to the size of the source instance, I.

To see this, consider a simple example composed of ded m_{d_1} above:

$$m_{d_1} : \forall x : S_1(x) \rightarrow (\exists y : T_1(x, y)) \vee T_2(x, x)$$

Given $I = \{S_1(a), S_1(b), S_1(c)\}$, the universal model set for the ded contains eight different solutions, each one corresponding to one way to choose among the branches in the conclusions of m_{d_1} for a tuple in S_1:

$$\begin{aligned} \mathbf{J} = \{ \quad & \{T_1(a, N_1), T_1(b, N_2), T_1(c, N_3)\}, \quad \{T_1(a, N_1), T_1(b, N_2), T_2(c, c)\} \\ & \{T_1(a, N_1), T_2(b, b), T_1(c, N_3)\}, \quad \{T_1(a, N_1), T_2(b, b), T_2(c, c)\} \\ & \{T_2(a, a), T_1(b, N_2), T_1(c, N_3)\}, \quad \{T_2(a, a), T_1(b, N_2), T_2(c, c)\} \\ & \{T_2(a, a), T_2(b, b), T_1(c, N_3)\}, \quad \{T_2(a, a), T_2(b, b), T_2(c, c)\} \quad \} \end{aligned}$$

In the general case, for source instances of size n we may have universal model sets of $O(k^n)$, where k depends on the number of disjunctions in ded conclusions. Therefore, one of the technical challenges posed by this problem is to tame this exponential complexity.

6 Correctness

We need to introduce a few preliminary notions. A crucial requirement about our rewriting algorithm is that the result of executing the source-to-target mapping is "the same" as the one that we would obtain if the source-to-ontology mapping were to be executed. Intuitively, we mean that a solution produced by the source-to-target mapping induces a solution for the source-to-ontology mapping when applying the view definitions.

To be more precise, consider the source-to-ontology mapping scenario: $\mathcal{M}_{SV} = \{\mathbf{S}, \mathbf{V}, \Sigma_{SV}, \Sigma_V\}$. For each source instance I, assume there exists a solution J_V for I and $\mathcal{M}_{SV}$ that complies with the view definitions in Σ_V (i.e. there exists an instance J_T of schema $\mathbf{T}$ such that $J_V = \Upsilon_V(J_T)$). Figure 3a and 3b show one example of I and J_V.

a. Source instance I

S-WorkerGrades(*1*, *2012*, *7*, *100*) *S-Stats*(*1*, *John*, *7*, *8*)
S-WorkerGrades(*1*, *2013*, *8*, *200*)

b. Ontology instance $J_{T'}$

Average(1, John)	Worker(1, John)
Evaluation(1, 2012)	Evaluation(1, 2013)
PositiveEval(1, 2012, 100)	PositiveEval(1, 2013, 200)
Year(2012)	Year(2013)

c. Target instance J_T

Employees(*1*, *John*) *Evaluations*(*1*, *2012*) *Evaluations*(*1*, *2013*)
PositiveEvals(*1*, *2012*, *100*) *PositiveEvals*(*1*, *2013*, *200*) *Warned*(*1*, N_1)

Figure 3: Source, ontology, and target instances.

We compute our rewriting, and obtain a new source-to-target scenario: $\mathcal{M}_{ST} = \{\mathbf{S}, \mathbf{T}, \Sigma_{ST}, \Sigma_T\}$, where we assume that Σ_{ST} and Σ_T are sets of deds. We may run $\mathcal{M}_{ST}$ on I to obtain solutions under the form of target instances. To any target instance J_T of this kind, we may apply the view definitions in Υ_V in order to obtain an instance of $\mathbf{V}$, $J_V = \Upsilon_V(J_T)$.

Our first intuition about the correctness of the algorithm is that the rewritten source-to-target scenario, $\mathcal{M}_{ST}$, should generate solutions, i.e. target instances that are guaranteed to generate views that, in turn, are solutions for the original source-to-ontology scenario, $\mathcal{M}_{SV}$. More precisely:

Definition 3 (Correct Rewriting) *Given a source-to-ontology scenario $\mathcal{M}_{SV} = \{\boldsymbol{S}, \boldsymbol{V}, \Sigma_{SV}, \Sigma_V\}$ with view definitions Υ_V, we say that the source-to-target rewritten scenario $\mathcal{M}_{ST} = \{\boldsymbol{S}, \boldsymbol{T}, \Sigma_{ST}, \Sigma_T\}$ with deds is a* correct rewriting *of $\mathcal{M}_{SV}$ if, for each instance I of the source database, whenever a universal model set $\boldsymbol{J} = \{J_0, \ldots, J_n\}$ for I and M_{ST} exists, then for each $J_i \in \boldsymbol{J}$, $\Upsilon_V(J_i)$ is also a solution for I and the original scenario $\mathcal{M}_{SV}$.*

The meaning of this definition is illustrated in Figure 4.

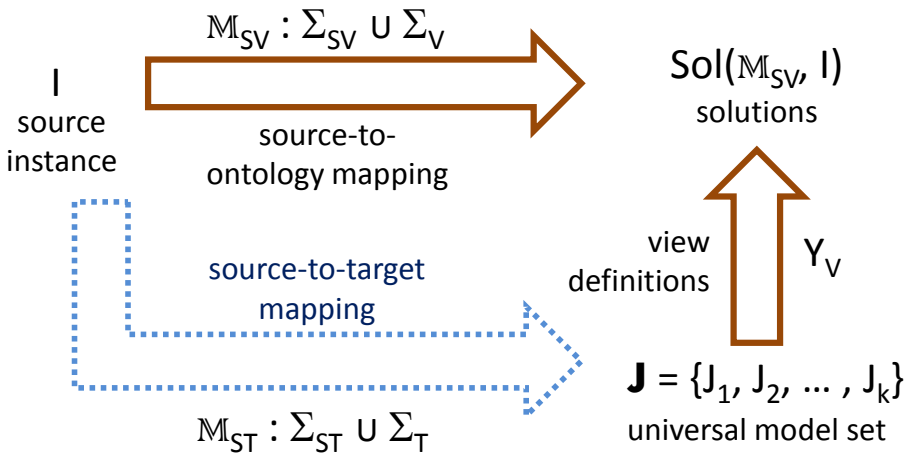


Figure 4: Correctness Diagram.

Figure 3c reports a correct target solution for I (N_1 is a labeled null). Note that $\Sigma_V(J_T)$ is exactly the ontology instance $J_{T'}$ in Figure 3b. A different font is used for entity and relationship types in the ontology instance.

7 The Rewriting Algorithm

In the following, we always assume that the input mapping captures all of the semantics from the ontology level. This means that all referential constraints implicit in the ontology (i.e. ontology tgds) have to be made explicit and properly encoded into the mapping dependencies [9]. In particular, whenever a relational atom $V(\bar{x})$ appears in the conclusion of a mapping dependency m and there is an ontology tgd $e : V(\bar{x}) \rightarrow \psi(\bar{x})$, we replace $V(\bar{x})$ by $\psi(\bar{x})$ in m. We restrict the textual integrity constraints to be key constraints and functional dependencies, and assume they are expressed as logical dependencies (i.e. egds) over the views (an automatic OCL-to-logic translation is proposed in [23]). Figure 5 shows the complete set of mapping dependencies Σ_{SV} for our running example.

$$
\begin{aligned}
m_0 &: \forall id, yr, gr, sinc, name, maxgr, mingr : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr), \\
&\qquad maxgr > 4, mingr < 9 \rightarrow \mathsf{Average}(\mathsf{id}, \mathsf{name}), \mathsf{Worker}(\mathsf{id}, \mathsf{name}) \\
m_1 &: \forall id, yr, gr, sinc, name, maxgr, mingr : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr), \\
&\qquad mingr \geq 9 \rightarrow \mathsf{Outstanding}(\mathsf{id}, \mathsf{name}), \mathsf{Worker}(\mathsf{id}, \mathsf{name}) \\
m_2 &: \forall id, yr, gr, sinc, name, maxgr, mingr : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \mathsf{Problematic}(\mathsf{id}, \mathsf{name}), \mathsf{Worker}(\mathsf{id}, \mathsf{name}) \\
m_3 &: \forall id, yr, gr, sinc : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), gr \geq 5 \rightarrow \exists name : \mathsf{PositiveEval}(\mathsf{id}, \mathsf{yr}, \mathsf{sinc}), \\
&\qquad \mathsf{Evaluation}(\mathsf{id}, \mathsf{yr}), \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \mathsf{Year}(\mathsf{yr}) \\
m_4 &: \forall id, yr, gr, sinc : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), gr < 5 \rightarrow \exists name : \mathsf{NegativeEval}(\mathsf{id}, \mathsf{yr}), \\
&\qquad \mathsf{Evaluation}(\mathsf{id}, \mathsf{yr}), \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \mathsf{Year}(\mathsf{yr})
\end{aligned}
$$

Figure 5: Source-to-ontology mapping.

Our algorithm generates:

(a) a new set of source-to-target tgds, Σ_{ST};

(b) a set of target dependencies, Σ_T. This latter set will contain:

  (b1) a set of target deds that model egds over the ontology schema. However, it may also incorporate other constraints that were not in the input. More precisely:

  (b2) a set of target deds, i.e. deds defined over the symbols in the target only;

  (b3) a set of denial constraints.

Denial constraints are crucial in our approach. Recall from Section 3 that a denial constraint is a dependency of the form $\forall \overline{x}\big(\varphi(\overline{x}) \rightarrow \bot\big)$. We use these to express the fact that some tuple configurations in the target are not compatible with the view definitions, and therefore should cause a failure in the mapping process. In other words, we are expressing part of the semantics of negations that comes with view definitions, in the form of failures of the data exchange process. This prevents our algorithm from being complete, as stated in Theorem 1, but guarantees that it is sound.

Given our input source-to-ontology mapping scenario, $\mathcal{M}_{SV} = \{\mathbf{S}, \mathbf{V}, \Sigma_{SV}, \Sigma_V\}$, our approach is to progressively rewrite dependencies in Σ_{SV} and Σ_V in order to remove view symbols, and replace them with target relations. To do this, we apply a number of transformations that guarantee that the rewritten mapping yields equivalent results wrt to input one, in the sense discussed in Section 4.

Algorithm 1 reports the pseudocode of our unfolding algorithm *UnfoldDependencies*. To define the algorithm, we use the standard unfolding algorithm for (positive) conjunctive views, *unfoldView* [25], as a building block.

The main intuition behind the algorithm is easily stated: it works with a set of dependencies, called Σ, initialized as $\Sigma_{SV} \cup \Sigma_V$, and progressively transforms this set until a fixpoint is reached. Note that it always terminates, since we assume the view definitions are not recursive. The algorithm employs four main transformations in order to remove derived atoms from the dependencies of Σ:

Transformation 1: First, whenever a positive derived atom $L(\bar{x}_i)$ is found in a dependency d, the algorithm uses the standard view unfolding algorithm as a building block in order to replace $L(\bar{x}_i)$ by its view definitions. The alternative definitions that may exist for a single view are handled in parallel. Therefore, the unfolding algorithm

Algorithm 1 *UnfoldDependencies*$(\Sigma_{SV}, \Sigma_V, \Upsilon_V)$

$\Sigma := \Sigma_{SV} \cup \Sigma_V$
repeat
  for all $d \in \Sigma$ **do**
    // Transformation 1.
    if d contains a positive derived atom L **then**
      for all view definition v_i of L in Υ_V **do**
        $\Sigma := \Sigma \cup \{unfoldView(L, d, v_i)\}$
      end for
      $\Sigma := \Sigma - \{d\}$
    end if
    // Transformation 2.
    if d is a ded containing a negative derived atom $\neg L(\bar{x}_i, \bar{y}_i)$ in $\psi_j(\bar{x}, \bar{y}_j)$ **then**
      let TGD_k be a new relation symbol
      $d := \phi(\bar{x}) \rightarrow \ldots \vee (\psi_j(\bar{x}, \bar{y}_j) - \{\neg L(\bar{x}_i, \bar{y}_i)\}) \cup \{TGD_k(\bar{x}_i, \bar{y}_i)\} \vee \ldots$
      $d^1 := TGD_k(\bar{x}_i, \bar{y}_i) \wedge L(\bar{x}_i, \bar{y}_i) \rightarrow \bot$
      $\Sigma := \Sigma \cup \{d^1\}$
    end if
    // Transformation 3.
    if d is a denial $\phi(\bar{x}) \rightarrow \bot$ containing a negative atom $\neg L(\bar{x}_i)$ in $\phi(\bar{x})$ **then**
      $d := \phi(\bar{x}) - \{\neg L(\bar{x}_i)\} \rightarrow L(\bar{x}_i)$
    end if
    // Transformation 4.
    if d is a ded containing a negative atom $\neg L(\bar{x}_i)$ in $\phi(\bar{x})$ **then**
      $d := \phi(\bar{x}) - \{\neg L(\bar{x}_i)\} \rightarrow \psi_1(\bar{x}, \bar{y}_1) \vee \ldots \vee \psi_n(\bar{x}, \bar{y}_n) \vee L(\bar{x}_i)$
    end if
  end for
until fixpoint
Σ_{ST} := the set of s-t deds in Σ
Σ_T := the set of target deds and denials in Σ

replaces dependency d with a set of dependencies $\{d'_1, d'_2, \ldots\}$, where each d'_i is like d after replacing $L(\bar{x}_i)$ by one of its definitions. To see an example, consider tgds m_0 and m_2, and views Average and Problematic:

$$\begin{aligned}
m_0 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr > 4, mingr < 9 \rightarrow \mathsf{Average}(\mathsf{id}, \mathsf{name}), \mathsf{Worker}(\mathsf{id}, \mathsf{name}) \\
m_2 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \mathsf{Problematic}(\mathsf{id}, \mathsf{name}), \mathsf{Worker}(\mathsf{id}, \mathsf{name}) \\
v_5 &: \mathsf{Problematic}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \textit{Penalized}(id, year) \\
v_6 &: \mathsf{Problematic}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{PositiveEval}(\mathsf{id}, \mathsf{year}, \mathsf{sinc}) \\
v_8 &: \mathsf{Average}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{Outstanding}(\mathsf{id}, \mathsf{name}), \\
&\qquad \neg\mathsf{Problematic}(\mathsf{id}, \mathsf{name})
\end{aligned}$$

Standard unfolding with v_8 changes m_0 as follows:

$$\begin{aligned}
m_0 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr > 4, mingr < 9 \rightarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{Outstanding}(\mathsf{id}, \mathsf{name}), \\
&\qquad \neg\mathsf{Problematic}(\mathsf{id}, \mathsf{name})
\end{aligned}$$

Standard unfolding with v_5 and v_6, respectively, changes m_2 as follows:

$$\begin{aligned}
m_{2a} &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \exists year' : \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \textit{Penalized}(id, year') \\
m_{2b} &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \exists year', sinc' : \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{PositiveEval}(\mathsf{id}, \mathsf{year}', \mathsf{sinc}')
\end{aligned}$$

Note that a single unfolding step might not be enough to fully remove all positive derived atoms, so successive applications of this first transformation may be required. In the example above, unfolding m_0, m_{2a} and m_{2b} with view Worker yields:

$$\begin{aligned}
m_0 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr > 4, mingr < 9 \rightarrow \textit{Employees}(id, name), \neg\mathsf{Outstanding}(\mathsf{id}, \mathsf{name}), \\
&\qquad \neg\mathsf{Problematic}(\mathsf{id}, \mathsf{name}) \\
m_{2a} &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \exists year' : \textit{Employees}(id, name), \textit{Penalized}(id, year') \\
m_{2b} &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \exists year', sinc' : \textit{Employees}(id, name), \neg\mathsf{PositiveEval}(\mathsf{id}, \mathsf{year}', \mathsf{sinc}')
\end{aligned}$$

Transformation 2: The second, and most important transformation, handles negated view atoms $\neg L(\bar{x}_i, \bar{y}_i)$ in tgd conclusions, e.g. Outstanding and Problematic in m_0 and PositiveEval in m_{2b}; we cannot directly unfold a negated derived atom of the conclusion in order to have an equivalent tgd; we need a way to express more appropriately the intended semantics, i.e, the fact that the tgd should be fired only if it is not possible to satisfy $L(\bar{x}_i, \bar{y}_i)$; to express this, we replace the negated atom from the conclusion (let us focus on m_0 and Outstanding for now) with a new relation symbol $TGD_i(\bar{x}_i, \bar{y}_i)$

$$\begin{aligned}
m_0^1 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr > 4, mingr < 9 \rightarrow \textit{Employees}(id, name), TGD_0(id, name), \\
&\qquad \neg\mathsf{Problematic}(\mathsf{id}, \mathsf{name})
\end{aligned}$$

and introduce a new dependency d^1, which states that d should fire only if it is not possible to satisfy $L(\bar{x}_i, \bar{y}_i)$, by means of a denial constraint:

$$m_0^2 : TGD_0(id, name), \mathsf{Outstanding}(\mathsf{id}, \mathsf{name}) \rightarrow \bot$$

Note that since a tgd may have more than one negated atom in the conclusion, the second transformation may have to be applied multiple times. The full result of the transformation when successively applied to m_0, m_{2a} and m_{2b} is the following:

$$\begin{aligned}
m_0^1 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr > 4, mingr < 9 \rightarrow \textit{Employees}(id, name), TGD_0(id, name) \\
m_0^2 &: TGD_0(id, name), \mathsf{Outstanding}(\mathsf{id}, \mathsf{name}) \rightarrow \bot \\
m_0^3 &: TGD_0(id, name), \mathsf{Problematic}(\mathsf{id}, \mathsf{name}) \rightarrow \bot \\
m_{2a}^1 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \exists year' : \textit{Employees}(id, name), \textit{Penalized}(id, year') \\
m_{2b}^1 &: \textit{S-WorkerGrades}(id, yr, gr, sinc), \textit{S-Stats}(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \exists year', sinc' : \textit{Employees}(id, name), TGD_1(id, year', sinc') \\
m_{2b}^2 &: TGD_1(id, year, sinc), \mathsf{PositiveEval}(\mathsf{id}, \mathsf{year}, \mathsf{sinc}) \rightarrow \bot
\end{aligned}$$

Transformation 3: The third transformation consists of moving negated atoms of the form $\neg L(\bar{x}_i)$ in the premise of a denial constraint d to its conclusion, in order to remove the negation. To see an example of this, we advance in the rewriting of m_0^2; transformation 1 needs to be applied again in order to unfold the Outstanding atom:

$$m_0^2 : TGD_0(id, name), \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{NegativeEval}(\mathsf{id}, \mathsf{year}), \\ \neg Warned(id, date) \rightarrow \bot$$

However, the negative atoms may be moved easily to the conclusion, to yield a target dtgd:

$$m_0^2 : TGD_0(id, name), \mathsf{Worker}(\mathsf{id}, \mathsf{name}) \rightarrow \exists year, date : \mathsf{NegativeEval}(\mathsf{id}, \mathsf{year}) \\ \vee Warned(id, date)$$

To complete the rewriting of m_0^2, the unfolding algorithm would keep on applying transformations 1 and 2.

Transformation 4: The fourth and final transformation is a variation of transformation 3 that is applied to deds. The only difference is that the atoms being moved from the premise are disjuncted to the current contents of the conclusion instead of replacing it.

The complete rewriting of the running example is reported in B.

7.1 Correctness Result

We are now ready to state our main result about the correctness of the rewriting algorithm. Before we do that, we should make more precise the schemas that are involved in the translation. We start with a target schema, $\mathbf{T}$, but during the rewriting we enrich it with new relation symbols, $TGD_0, TGD_1, \ldots$, in order to be able to correctly specify denials. We call the resulting schema $\mathbf{T}'$.

Theorem 2 (Correctness) *Given a source-to-ontology scenario $\mathcal{M}_{SV} = \{\boldsymbol{S}, \boldsymbol{V}, \Sigma_{SV}, \Sigma_V\}$ with non-recursive view definitions Υ_V, then:*

(a) algorithm UnfoldDependencies always terminates;

(b) when it does not fail, it computes a correct source-to-target rewritten scenario with deds $\mathcal{M}_{ST'} = \{\boldsymbol{S}, \boldsymbol{T'}, \Sigma_{ST'}, \Sigma_{T'}\}$, where $\boldsymbol{T'}$ is obtained from $\boldsymbol{T}$ by enriching it with a finite set of new relation symbols $TGD_0, TGD_1, \ldots$.

8 A Restricted Case

Theorem 2 shows that Algorithm 1 is correct. However, we also know that it may incur significant scalability issues, that we discuss in Section 9. This leaves us with a crucial question: is it possible to find a view-definition language that is at the same time more expressive than plain conjunctive queries, and computes correct rewritings in terms of embedded dependencies, i.e. tgds, egds, and standard denial constraints only?

In this section, we show that such a view-definition language exists, and corresponds to non-recursive Datalog with a limited negation. To be more precise, we limit negation in such a way that: (*i*) we disallow some pathological patterns within view definitions with negations; (*ii*) keys and functional dependencies — i.e. egds — are defined only for views whose definition does not depend on negated atoms.

Definition 4 (Negation-Safe View Language) *Given a set of non-recursive view definitions, Υ_V, we say that these are negation-safe if the following occur:*

1. *there is no view V_i that negatively depends on a view V_j that in turn negatively depends on two negated atoms;*
2. *keys and functional dependencies are defined only for views whose definitions do not contain negated atoms.*

In essence, item 1 above disallows very specific view-definition patterns, like the one below:

$$v_1 : \mathsf{V}_1(\mathsf{x}, \mathsf{y}) \Leftarrow T_1(x, y), \neg\mathsf{V}_2(\mathsf{x}, \mathsf{y}) \\ v_2 : \mathsf{V}_2(\mathsf{x}, \mathsf{y}) \Leftarrow T_2(x, y), \neg\mathsf{V}_3(\mathsf{x}, \mathsf{y}), \neg\mathsf{V}_4(\mathsf{x}, \mathsf{y}) \\ \ldots$$

Item 1 prohibits the definition of keys on views $\mathsf{V}_1, \mathsf{V}_2$ that contain negated views in their definitions. We can show that the condition in Definition 4 is a sufficient condition that guarantees that Algorithm 1 returns a set of embedded dependencies, and does not generate deds.

Theorem 3 (Restriction) *Given a source-to-ontology scenario $\mathcal{M}_{SV} = \{\boldsymbol{S}, \boldsymbol{V}, \Sigma_{SV}, \Sigma_V\}$ with view definition Υ_V, assume Υ_V conforms to the restrictions in Definition 4. Call $\mathcal{M}^{emb}_{ST'} = \{\boldsymbol{S}, \boldsymbol{T'}, \Sigma_{ST'}, \Sigma_{T'}\}$, the source-to-target rewritten scenario computed by algorithm UnfoldDependencies, where $\boldsymbol{T'}$ is obtained from $\boldsymbol{T}$ by enriching it with a finite set of new relation symbols $TGD_0, TGD_1, \ldots$. Then $\mathcal{M}^{emb}_{ST'}$ only contains embedded dependencies (i.e. tgds, egds, and denial constraints).*

Theorem 3 guarantees that, under the conditions of Definition 4, the rewritten source-to-target mapping is a set of standard tgds, egds, and denial constraints. This has important implications on the scalability of the data-exchange process, as we discuss in the next section.

9 The Chase Engine

Once we have computed our source-to-target mapping, we can concretely attempt the actual data exchange, and move data from the source database to the target. The standard way to do this corresponds to running the well known *chase* [8] procedure, i.e. an operational semantics for embedded dependencies that we discuss in the following.

9.1 The Chase

Given a vector of variables $\overline{v}$, an *assignment* for $\overline{v}$ is a mapping $a : \overline{v} \rightarrow \text{CONST} \cup \text{NULLS}$ that associates with each universal variable a constant in CONST, and with each existential variable either a constant or a labeled null. Given a formula $\phi(\overline{x})$ with free variables $\overline{x}$, and an instance I, we say that I *satisfies* $\phi(a(\overline{x}))$ if $I \models \phi(a(\overline{x}))$, according to the standard notion of logical entailment.

Of the many variants of the chase, we consider the *naive chase* [26]. We first introduce the notions of *chase steps* for tgds, egds, and denial constraints, and then the notions of a chase sequence and of a chase result.

Chase Step for Tgds: Given instances I, J, a tgd $\phi(\overline{x}) \rightarrow \exists \overline{y}(\psi(\overline{x}, \overline{y}))$ is fired for all assignments a such that $I \models \phi(a(\overline{x}))$; to fire the tgd, a is extended to $\overline{y}$ by injectively assigning to each $y_i \in \overline{y}$ a fresh null, and then adding the facts in $\psi(a(\overline{x}), a(\overline{y}))$ to J. To give an example, consider the following tgd:

$$m.\ \mathit{Driver}(\mathit{name}, \mathit{plate}) \rightarrow \exists \mathit{Bdate}, \mathit{CarId}\colon \mathit{Person}(\mathit{name}, \mathit{BDate}, \mathit{CarId}), \mathit{Car}(\mathit{CarId}, \mathit{plate})$$

During the chase, the source tuple *Driver(Jim, abc123)* will generate the two target tuples $\mathit{Person}(Jim, N_1, C1)$, and *Car(C1, abc123)*, where N_1, C_1 are fresh labeled nulls.

Chase Step for Egds: To chase an egd $\forall \bar{x} : \phi(\bar{x}) \rightarrow x_i = x_j$ over an instance J, for each assignment a such that $J \models \phi(a(\overline{x}))$, if $a(x_i) \neq a(x_j)$, the chase tries to equate the two values. We distinguish two cases: (i) both $a(x_i)$ $a(x_j)$ are constants; in this case, the chase procedure *fails*, since it attempts to identify two different constants; (ii) at least one of $a(x_i)$, $a(x_j)$ is a null, say $a(x_i)$; in this case chasing the egd generates a new instance J' obtained from J by replacing all occurrences of $a(x_i)$ by $a(x_j)$. To give an example, consider egd e_1:

$$e_1.\ \mathit{Person}(\mathit{name}, b, c), \mathit{Person}(\mathit{name}, b', c') \rightarrow (b = b') \wedge (c = c')$$

Assume two tuples have been generated by chasing the tgds, $\mathit{Person}\ (Jim, 1980, N_4)$, $\mathit{Person}\ (Jim, N_5, N_6)$, chasing the egd has two different effects: (i) it replaces nulls by constants; in our example, it equates N_5 to the constant 1980, based on the same value for the key attribute, *Jim*; (ii) on the other side, the chase might equate nulls; in our example, it equates N_4 to N_6, to generate a single tuple $\mathit{Person}(Jim, 1980, N_4)$.

Chase Step for Denial Constraints: Denial constraints can only generate failures. More specifically, the chase of a denial constraint $\forall \bar{x} : \phi(\bar{x}) \rightarrow \bot$ over an instance J fails whenever there exists an assignment a such that $J \models \phi(a(\overline{x}))$

Given a mapping scenario $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma_{ST}, \Sigma_T)$ and instance I, a *chase sequence* is a sequence of instances $J_0 = I, J_1, \ldots, J_k \ldots$, such that each J_i is generated by a chase step with $\Sigma_{ST} \cup \Sigma_T$ over J_{i-1}. The *chase* of $\Sigma_{ST} \cup \Sigma_T$ is an instance J_m such that no chase step is applicable. Notice that the chase may not terminate [8]. This may happen, for example, in the case of recursive target tgds. However, if it terminates, then J_m is a solution for $\mathcal{M}$ and I, called a *canonical solution*.

Any canonical solution is a universal solution [8]. Since all solutions obtained by using the naive chase are equal up to the renaming of nulls, we often speak of *the* canonical universal solution.

9.2 A Greedy Chase

For the purpose of this work, we adopt the chase engine developed within the LLUNATIC project [12, 13], that is freely available.[2] The chase engine was developed to guarantee high scalability, even for large sets of embedded dependencies, and large source instances.

Therefore, we expect that the data-exchange step can be completed quite efficiently under the conditions of Definition 4 and Theorem 3, i.e. when the rewriting algorithm returns a set of standard embedded dependencies.

Things change quite dramatically when the rewriting algorithm returns a set of deds. As we noticed in Section 5, deds have a perverse effect on the complexity of computing solutions. Given an instance I, a set of deds may have a number of solutions over I that is exponential in the size of I.

Intuitively, the chase also changes. In fact, the chase of deds generates *chase trees*, not chase sequences. Consider the following example, where we are given two deds:

$$m_{d_1} : \forall x : S_1(x) \rightarrow (\exists y : T_1(x, y)) \vee (T_2(x, x))$$
$$m_{d_2} : \forall x : S_2(x) \rightarrow (\exists y : T_3(x, y), T_3(y, x)) \vee (\exists z : T_4(x, z))$$

We start chasing these on source instance $I = \{S_1(a), S_2(b)\}$. A first assignment $a(x) =$ *'a'* such that $I \models S_1(a(x))$ is found, and therefore we may fire m_{d_1}. However, two alternative target instances may be generated, namely $J_1 = \{T_1(a, N_1)\}$ and $J_2 = \{T_2(a, a)\}$. These need to be considered in parallel, and therefore a chase tree rooted at $J_0 = \emptyset$, i.e. the empty target instance, with children J_1, J_2 is built. To proceed with the chase, we need to inspect every leaf, and apply successive chase steps. This happens with assignment $a(x) =$ *'b'*, according to which the premise of the second ded is satisfied by I. It is easy to see that we have two different ways to satisfy the ded, and therefore we end up with a chase tree with four leaves, each of which is a solution for this simple scenario. These, together, form a universal model set for the deds, as follows:

$$\mathbf{J} = \{ \quad \{T_1(a, N_1), T_3(b, N_2), T_3(N_2, b)\}, \quad \{T_2(a, a), T_3(b, N_2), T_3(N_2, b)\},$$
$$\{T_1(a, N_1), T_4(b, N_4)\}, \quad \{T_2(a, a), T_4(b, N_5)\} \quad \}$$

Recall that there are cases in which the size of the chase tree is exponential in the size of the input instance I. As a consequence, there is little hope that we are able to perform this parallel chase in a scalable way.

Recall, however, that our rewriting algorithm follows a best-effort approach. Along the same lines, we may consider giving up the idea of generating the entire tree, and rather concentrate on some of its branches, following a greedy strategy. To be more precise, we notice that the four leaves of the chase tree correspond each to the canonical solution of one of the following four sets of (standard) tgds:

$$\Sigma_{11}: \quad m_{11} : \forall x : S_1(x) \rightarrow (\exists y : T_1(x, y))$$
$$m_{21} : \forall x : S_2(x) \rightarrow (\exists y : T_3(x, y), T_3(y, x))$$

$$\Sigma_{12}: \quad m_{11} : \forall x : S_1(x) \rightarrow (\exists y : T_1(x, y))$$
$$m_{22} : \forall x : S_2(x) \rightarrow (\exists z : T_4(x, z))$$

$$\Sigma_{21}: \quad m_{12} : \forall x : S_1(x) \rightarrow T_2(x, x)$$
$$m_{21} : \forall x : S_2(x) \rightarrow (\exists y : T_3(x, y), T_3(y, x))$$

$$\Sigma_{22}: \quad m_{12} : \forall x : S_1(x) \rightarrow T_2(x, x)$$
$$m_{22} : \forall x : S_2(x) \rightarrow (\exists z : T_4(x, z))$$

For example, Σ_{11} generates those solutions that were generated by the chase of m_{d_1}, m_{d_2} along those branches of the chase tree in which the first conjunct of both deds was always chosen. Similarly for the others.

We call these the *greedy scenarios* associated with a mapping scenario with deds. Greedy scenarios do not generate all of the canonical solutions associated with a mapping scenario with ded. In fact, they are not able to capture the chase strategies in which the same ded is fired according to the first conjunct at some step, and according to another conjunct at a following step. However, their canonical solutions can be computed in a scalable way.

This justifies our chase strategy with deds:

(*i*) given a mapping scenario $\mathcal{M}^{ded}$ with a set of deds Σ^{ded}, we generate the associated greedy scenarios, $\mathcal{M}_0^{emb}, \mathcal{M}_1^{emb}, \ldots, \mathcal{M}_n^{emb}$; each is obtained by picking a different combination of the conjuncts that are present in ded conclusions;

(*ii*) given an instance I, we start chasing the greedy scenarios, one by one, on I; as soon as we get a canonical solution J_i for greedy scenario $\mathcal{M}_i^{emb}$ and I, we return J_i and stop;

[2] `http://db.unibas.it/projects/llunatic`

(*iii*) if every greedy scenario fails on I, we fail and return no solution.

In the following section, we study the scalability of this approach.

10 Experiments

We implemented a prototype of our rewriting algorithm in Java. In order to execute the mappings, we used the free and highly scalable chase engine LLUNATIC [13]. We performed our experiments on an Intel core i7 machine with a 2.6 GHz processor, 8 GB of RAM, and running MacOSX. We used PostgreSQL 9.2.1 (x64 version) as the DBMS.

Scenarios We used three different datasets from which we derived a number of different scenarios:

(*a*) WORKERS is obtained by applying the unfolding algorithm to the source-to-ontology mapping scenario described in the B. This is a ded-based scenario with 3 source and 15 target tables. It contains 23 deds that generate 20 different greedy scenarios.

(*b*) Recall that scenarios with deds are chased by successively chasing their greedy versions. Since we are also interested in studying how each of these greedy scenarios (without deds) impacts performance, in our tests we also consider the first greedy scenario generated for WORKERS, and denote it by WORKERS-GREEDY-1. This has 10 st-tgds, 4 target tgds, 3 target egds, and 7 denial constraints.

(*c*) EMPLOYEES is a traditional schema mapping scenario based on the example proposed in [18]. It contains 2 source and 10 target tables, 9 st-tgds, 5 target tgds, 2 target egds, and 2 denial constraints.

(*d*) To study the impact of egds on the rewriting algorithm and on the chase, we also consider an egd-free version of EMPLOYEES, called EMPLOYEES NO-EGD.

(*e*) Finally, we want to test the scalability of the rewriting algorithm. For this purpose, we take a fully synthetic dataset, called SYNTHETIC. Based on this, we generated seven different scenarios, with a number of dependencies ranging from 50 to 30K dependencies.

Effectiveness To measure the effectiveness of our approach, we compared the size of the source-to-ontology mapping that users need to specify for the various scenarios, to the size of the actual source-to-target scenario generated by our rewriting. As a measure of the size of a scenario, we took the number of nodes and edges of the *dependency graph* [8], i.e. the graph in which each atom of a dependency is a node, and there is an edge from node n_1 to node n_2 whenever the corresponding atoms share a variable. Intuitively, the higher the complexity of this graph, the more complicated it is to express the mapping. Figure 6a reports the results for 5 scenarios. In all scenarios there was a considerable increase in the size of the dependency graph (up to 70%). This is a clear indication that in many cases our approach is more effective with respect to manually developing the source-to-target mapping.

Scalability of the Rewriting Algorithm The second set of experiments tests the scalability of our unfolding algorithm on mapping scenarios of a large size. Figure 6b summarizes results of these experiments on scenarios of increasing size. All source-to-ontology tgds in these scenarios have two source relations in the premise and two views in the conclusion. Each view definition has two positive target relational symbols and (if the view has negation) two negated view symbols. For each mapping scenario, 20% of the tgds have no negated atoms, the next 20% have 1 level of negation (i.e. negated atoms that do not depend in turn on other negations), the next 20% have 2 levels of negation, and so on, up to 4 levels of negations. The number of source relations in the mapping scenarios ranges from 10k to 60k, the number of view definitions ranges from 238k to 1428k, and the number of target relations ranges from 228k to 1368k. The reported times are the running times of the unfolding algorithm running in main memory, and do not include disk read and write times. The rewriting algorithm scales nicely to large scenarios.

Scalability of the Chase Our final goal is to study the scalability of the chase engine, i.e. how expensive it is to execute the source-to-target rewritten mapping. To do this, we first study the performance of the chase engine on schema mapping scenarios with no deds. This is important, since previous research [16, 17, 15] have shown that some of the existing chase engines hardly scale to large datasets. Figure 6c and 6d report the time needed to compute a solution for four of our scenarios. As expected, scenarios with no egds required lower computing times. However, in the case of egds the chase engine also scaled nicely to databases of 1 million tuples.

To test scenarios with deds, we developed the greedy-chase algorithm described in Section 9.2 on top of LLUNATIC. Recall that, given a mapping scenario with deds, we generate a set of greedy scenarios with embedded dependencies only. The first experiment in this context was to test how many of the 20 greedy scenarios associated to the WORKERS scenario do return a solution.

We first generated four different random source instances and in Figure 6e we report the results. The greedy algorithm generated a solution in all of the four cases. Then, we studied the possible failure conditions, and manually

crafted an instance with a high probability of triggering the denial constraints. With this fifth source instance, all of the 20 greedy scenarios failed. Notice that a solution still exists. However, this is not captured by the combinations of atoms in greedy scenarios, and would require the generation of the entire chase tree to be found.

Finally in Figure 6f we report scalability results for the greedy chase algorithm. For each execution we also report the number of greedy scenarios that the chase engine needed to run in order to reach a solution. As can be seen, the chase scales nicely, even with databases of 1 million of tuples. Spikes in computing times are due to the need to execute fewer scenarios before a solution is found. To the best of our knowledge, this is the first scalability result for the chase of disjunctive embedded dependencies.

11 Related Work

The standard view unfolding algorithm [25] has been used extensively in data integration as a tool for query answering. In such a setting, users pose queries over a set of heterogeneous sources through a single global schema, which provides a uniform view of all the sources. Mappings between the sources and the global schema are used to rewrite the users' queries in terms of the sources. One way to define these mappings is the so-called global-as-view approach (GAV), in which the global schema is defined as a view over the sources. With this kind of mapping, answering a query posed on the global schema usually reduces to unfolding the view definitions [14] (unless integrity constraints are present in the global schema, which makes answering harder [4]).

Another similar problem is that of accessing data through ontologies, in which users pose queries on an ontology that is defined on top of a set of databases; the ontology plays the role of global schema, and the databases play the role of data sources [21, 5]. The problem we address in this paper, however, is not about using view unfolding to answer queries, but to copy data into a target. As we have discussed in Section 7, standard view unfolding suffices only when the views that define the target conceptual schema in terms of the underlying database are plain conjunctive queries. In the presence of negation, copying data into the target gets more complicated, as negated atoms in mapping conclusions introduce new integrity constraints that standard view unfolding does not handle (intuitively, negated atoms must be kept false during all the process of copying data into the target).

A problem that relates to our use of view unfolding in mappings is that of mapping composition [10, 20]. Composing a mapping between schemas A and B with a mapping between schemas B and C produces a new mapping between A and C. In a sense, our application of view unfolding to the conclusion of a mapping can be seen as a kind of mapping composition; one in which the mapping between the source and the conceptual schema is composed with a second mapping that relates the conceptual schema with the underlying database (i.e. the views). However, mapping composition techniques take into account the direction of the mapping, that is, one can compose a mapping from A to B only with another mapping that goes from B to some C in order to get a mapping that goes from A to C. In our case, we have a mapping from the source to the conceptual schema and another one from the database to the conceptual schema, which cannot be directly composed.

The introduction of conceptual schemas into the mapping process has also been investigated in [1] with respect to a different problem, i.e. that of generating mappings between databases. Since we assume that source-to-ontology mappings are given as inputs, the techniques developed in [1] can be used as a preliminary step to simplify the mapping specification phase.

Another context where mappings involving conceptual schemas have been studied is that of Semantic Web ontologies; in particular, [24] proposes a technique that translates a set of correspondences between source and target ontologies into a set of SPARQL queries that can then be run against the data source to produce the target's data. Comparing with our approach, we assume that the given mapping is not just a set of correspondences, but a complete declarative mapping expressed as tgds, and we also take into account that the target's conceptual schema is a view of the underlying database.

Mappings between conceptual schemas have also been studied in [2], where the authors propose an approach for finding "semantically similar" associations between two conceptual schemas. These similar associations are then used to generate a mapping. This approach is complementary to ours in the sense that it could be used to generate a semantic-based mapping, which would then be rewritten using the algorithm we present in this paper.

12 Conclusion

This paper studies the problem of mapping data in the presence of ontology-based descriptions of the source and target data sources. It shows that employing an expressive view-definition language for the purpose of defining ontologies makes the rewriting process much more complicated than in the case of positive conjunctive views. The

paper develops an algorithm to automatically perform the rewriting when views are defined by means of non-recursive Datalog rules with negation. This, in turn, required the adoption of a very expressive mapping language involving disjunctive embedded dependencies.

To handle the increased complexity of this mapping language, we investigated restrictions to the view-definition language that may be handled using standard embedded dependencies (i.e. tgds and egds) for which efficient execution strategies exist. We conducted experiments on large databases and mapping scenarios to show the trade-off between expressibility of the view language and the efficiency of the data exchange step.

As future work, we plan to investigate the use of other execution strategies to perform the actual data-exchange to move data from the source to the target database rather than the greedy chase considered here. We would also like to analyze the applicability of our techniques to ontology based updating, seen as a parallel notion to the classical problem of ontology based querying.

References

[1] Y. An, A. Borgida, R.J. Miller, and J. Mylopoulos. A Semantic Approach to Discovering Schema Mapping Expressions. In *ICDE*, pages 206–215, 2007.

[2] Yuan An and Il-Yeol Song. Discovering semantically similar associations (sesa) for complex mappings between conceptual models. In *ER*, pages 369–382, 2008.

[3] C. Beeri and M.Y. Vardi. A Proof Procedure for Data Dependencies. *J.ACM*, 31(4):718–741, 1984.

[4] A. Calì, D. Calvanese, G. De Giacomo, and M. Lenzerini. Data integration under integrity constraints. *Inf. Syst*, 29(2):147–163, 2004.

[5] Diego Calvanese, Giuseppe De Giacomo, Maurizio Lenzerini, and Riccardo Rosati. View-based query answering in description logics: Semantics and complexity. *J. Comput. Syst. Sci.*, 78(1):26–46, 2012.

[6] S. Ceri, G. Gottlob, and L. Tanca. What you Always Wanted to Know About Datalog (And Never Dared to Ask). *IEEE TKDE*, 1(1):146–166, 1989.

[7] Alin Deutsch, Alan Nash, and Jeff Remmel. The chase revisited. In *PODS '08*, pages 149–158, 2008.

[8] R. Fagin, P.G. Kolaitis, R.J. Miller, and L. Popa. Data Exchange: Semantics and Query Answering. *TCS*, 336(1):89–124, 2005.

[9] R. Fagin, P.G. Kolaitis, A. Nash, and L. Popa. Towards a Theory of Schema-Mapping Optimization. In *PODS*, pages 33–42, 2008.

[10] Ronald Fagin, Phokion G. Kolaitis, Lucian Popa, and Wang Chiew Tan. Composing schema mappings: Second-order dependencies to the rescue. *ACM TODS*, 30(4):994–1055, 2005.

[11] F. Geerts, G. Mecca, P. Papotti, and D. Santoro. The LLUNATIC Data-Cleaning Framework. *PVLDB*, 6(9):625–636, 2013.

[12] F. Geerts, G. Mecca, P. Papotti, and D. Santoro. Mapping and Cleaning. In *ICDE*, pages 232–243, 2014.

[13] F. Geerts, G. Mecca, P. Papotti, and D. Santoro. That's All Folks! LLUNATIC Goes Open Source. *PVLDB*, 7(13):1565–1568, 2014.

[14] M. Lenzerini. Data integration: a Theoretical Perspective. In *PODS*, 2002.

[15] Bruno Marnette, Giansalvatore Mecca, Paolo Papotti, Salvatore Raunich, and Donatello Santoro. ++SPICY: an opensource tool for second-generation schema mapping and data exchange. *PVLDB*, 4(11):1438–1441, 2011.

[16] G. Mecca, P. Papotti, and S. Raunich. Core Schema Mappings: Scalable Core Computations in Data Exchange. *Inf. Syst*, 37(7):677–711, 2012.

[17] G. Mecca, P. Papotti, S. Raunich, and M. Buoncristiano. Concise and Expressive Mappings with +SPICY. *PVLDB*, 2(2):1582–1585, 2009.

[18] G. Mecca, G. Rull, D. Santoro, and E. Teniente. Semantic-Based Mappings. In *ER*, pages 255–269, 2013.

[19] R. J. Miller, L. M. Haas, and M. A. Hernandez. Schema Mapping as Query Discovery. In *VLDB*, pages 77–99, 2000.

[20] Alan Nash, Philip A. Bernstein, and Sergey Melnik. Composition of mappings given by embedded dependencies. *ACM Trans. Database Syst.*, 32(1):4, 2007.

[21] Antonella Poggi, Domenico Lembo, Diego Calvanese, Giuseppe De Giacomo, Maurizio Lenzerini, and Riccardo Rosati. Linking data to ontologies. *J. Data Semantics*, 10:133–173, 2008.

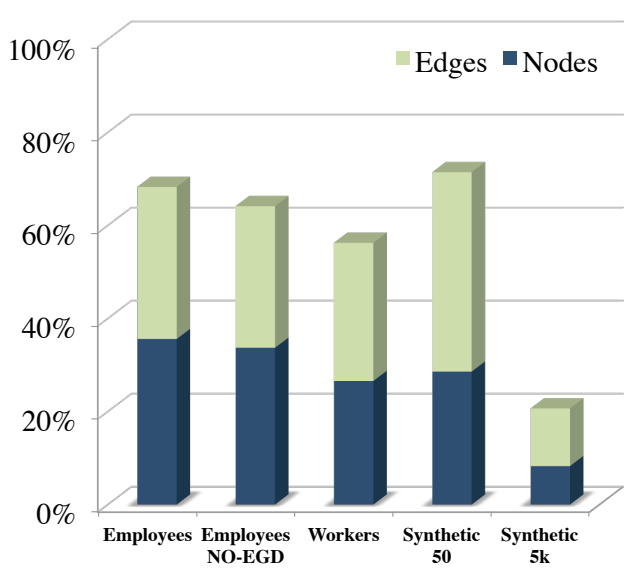
[22] L. Popa, Y. Velegrakis, R. J. Miller, M. A. Hernandez, and R. Fagin. Translating Web Data. In *VLDB*, pages 598–609, 2002.

[23] Anna Queralt and Ernest Teniente. Reasoning on uml class diagrams with ocl constraints. In *ER*, pages 497–512, 2006.

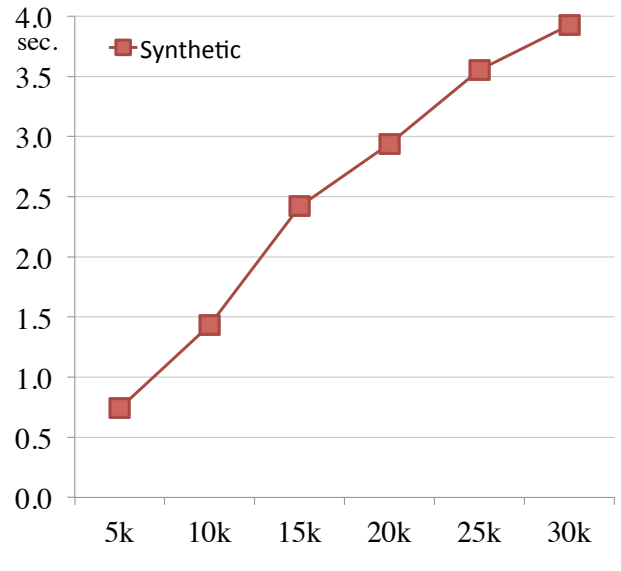
[24] Carlos R. Rivero, Inma Hernández, David Ruiz, and Rafael Corchuelo. Generating sparql executable mappings to integrate ontologies. In *ER*, pages 118–131, 2011.

[25] L. Sterling and E. Y. Shapiro. *The Art of Prolog: Advanced Programming Techniques*. MIT Press, 1994.

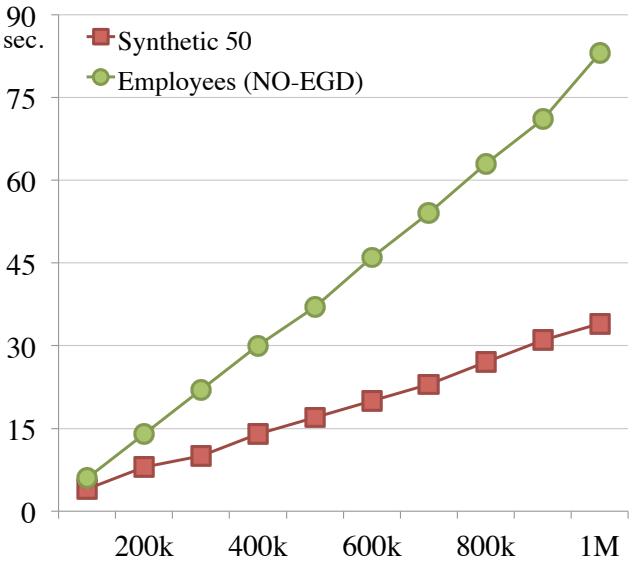
[26] B. ten Cate, L. Chiticariu, P. Kolaitis, and W. C. Tan. Laconic Schema Mappings: Computing Core Universal Solutions by Means of SQL Queries. *PVLDB*, 2(1):1006–1017, 2009.



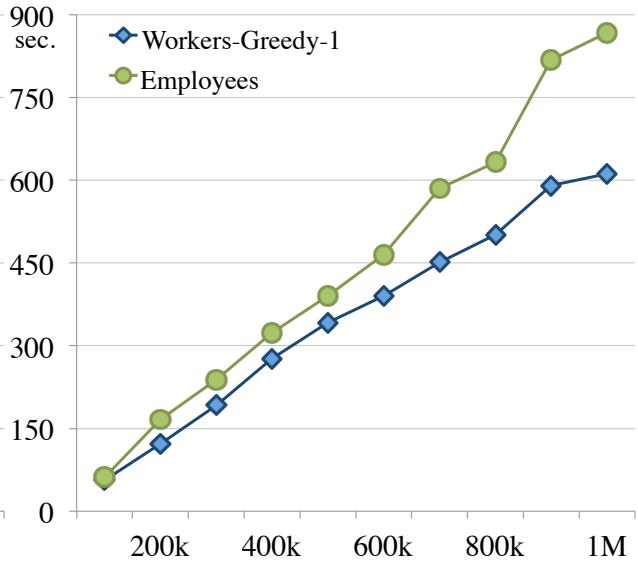
(a) Effectiveness



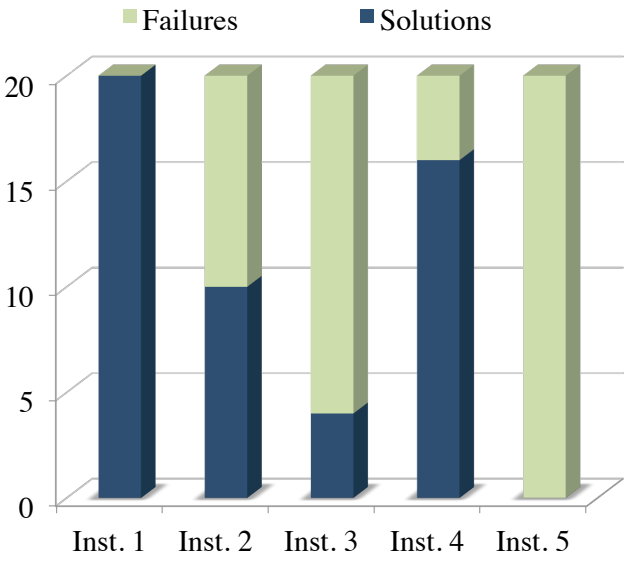
(b) Scalability of the rewriting algorithm



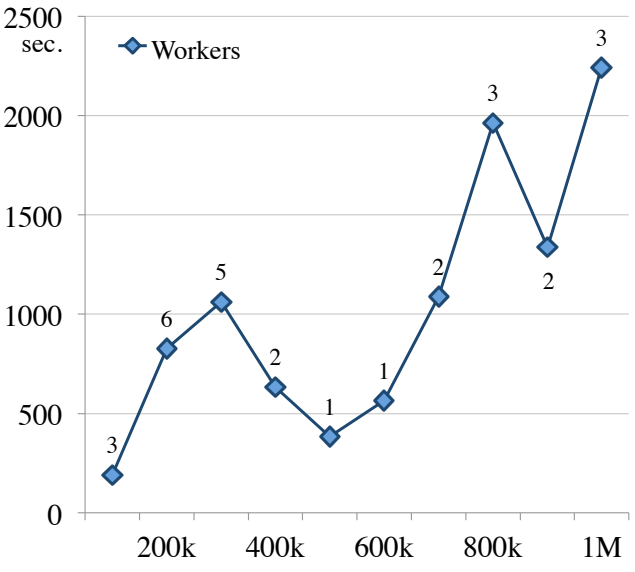
(c) Scalability of the chase algorithm



(d) Scalability of the chase for EGDs



(e) Results of the greedy scenarios



(f) Scalability of the chase for DEDs

Figure 6: Results of Experiments.

A Proofs of the Theorems

Theorem 1 *There exist a source-to-ontology mapping scenario $\mathcal{M}_{SV} = \{\boldsymbol{S},\ \boldsymbol{V},\ \Sigma_{SV}, \Sigma_V\}$ with view definition Υ_V, and an instance I, such that $\mathcal{M}_{SV}$ and I admit a universal solution $J_V \in \mathsf{USol}(\mathcal{M}_{SV}, I)$, and there exists no source-to-target scenario $\mathcal{M}_{ST}$ composed of embedded dependencies (tgds and egds) such that $\mathcal{M}_{ST}$ and I admit a solution J_T, and $J_V = \Upsilon(J_T)$.*

Proof: Consider the following scenario. The source database contains a single table, $S(A)$, the target database a single table, $T(A)$, and we have two views, $\mathsf{V}_1(\mathsf{A}), \mathsf{V}_2(\mathsf{A})$, defined as follows:

$$\Upsilon_V = \{ \quad \begin{array}{l} \mathsf{V}_1(\mathsf{x}) \Leftarrow T(x) \\ \mathsf{V}_2(\mathsf{x}) \Leftarrow T(x), \neg \mathsf{V}_1(\mathsf{x}) \end{array} \quad \}$$

The source-to-ontology mappings are the following (Σ_V is empty):

$$\Sigma_{SV} = \{ \quad \begin{array}{l} S(x) \rightarrow \mathsf{V}_1(\mathsf{x}) \\ S(x) \rightarrow \mathsf{V}_2(\mathsf{x}) \end{array} \quad \}$$

On instance $I = \{S(a)\}$, Σ_{SV} has a universal solution $J_V = \{\mathsf{V}_1(\mathsf{a}), \mathsf{V}_2(\mathsf{a})\}$.

We now prove that there exists no target instance J_T such that $\Upsilon_V(J_T) = J_V$. The view definitions in Υ_V are such that, for any target instance J, $\Upsilon_V(J)$ will not contain tuples $\mathsf{V}_1(\mathsf{c}), \mathsf{V}_2(\mathsf{c})$ for some constant c.

Since J_T does not exist, there is no source-to-target rewriting $\mathcal{M}_{ST}$ that may generate it as a universal solution for I, and the claim is proven. □

Theorem 2 *Given a source-to-ontology scenario $\mathcal{M}_{SV} = \{\boldsymbol{S},\ \boldsymbol{V},\ \Sigma_{SV}, \Sigma_V\}$ with non-recursive view definitions Υ_V, then:*

(a) algorithm UnfoldDependencies always terminates;

(b) when it does not fail, it computes a correct source-to-target rewritten scenario with deds $\mathcal{M}_{ST'} = \{\boldsymbol{S}, \boldsymbol{T'}, \Sigma_{ST'}, \Sigma_{T'}\}$, where $\boldsymbol{T'}$ is obtained from $\boldsymbol{T}$ by enriching it with a finite set of new relation symbols $TGD_0, TGD_1, \ldots$.

Proof: Let us first prove termination, and then correctness.

Termination — The proof of part a. depends on the fact that the view definitions in Υ_V are non-recursive by hypothesis. As a consequence, the set of view symbols, $\mathsf{V}_1, \mathsf{V}_2, \ldots, \mathsf{V}_k$ can be stratified, i.e. it can be partitioned in a sequence of subsets called *strata* such that any view that belongs to stratum i only depends directly or indirectly on those that appear in strata $1, 2, \ldots i-1$.

Algorithm *UnfoldDependencies* is composed of a main loop, and 4 different transformations (*Transformation 1.* to *4.*) that are applied to all dependencies in the current set. The loop stops when a fixpoint is reached. The effects of the various transformations are as follows:

- *Transformation 1.* unfolds view definition within a dependency d, i.e. it replaces a positively occurring view symbol by its definition; therefore, it removes a view symbol in stratum i and replaces it with target symbols or views that belong to strata up to $i-1$;
- *Transformation 2.* removes negatively derived atoms from dependency conclusions, and adds new dependencies;
- *Transformations 3.* and *4.* move negated atoms from a dependency premise to its conclusion.

Given a set of dependencies, Σ, we assign an integer score to it, based on the following function: the score for Σ is the sum of the scores for its dependencies. For each dependency, it is the sum of the scores of its atoms that contain view symbols. With a positive atom $\mathsf{V}(\bar{x}, \bar{y})$ it is associated an integer score k^i, where i is the stratum of V. With a negative atom $\neg\mathsf{V}(\bar{x}, \bar{y})$ it is associated an integer score $k^i + 1$, where i is again the stratum of V. It remains to define the value of k. Call n the maximum number of view symbols that appear in the body of a view definition of Υ_V. Then $k = n + 1$.

It is easy to see that the four transformations monotonically decrease the score of Σ. In fact:

- *Transformation 1.* replaces positive view atoms from stratum i by less than k view atoms that belong at most to stratum $i-1$;

- *Transformation 2.* removes a negated atom of stratum i from d, and introduces a new dependency d^1 that (only) contains a positive atom of the same stratum;
- *Transformations 3.* and *4.* replace a negated atom of stratum i within d by a positive atom of the same stratum in d^1.

Since each iteration of the cycle monotonically reduces the score of Σ, and this is initially finite, then the number of iterations is bounded, and the algorithm terminates.

Correctness — To prove part b., i.e. that the rewritten scenario is correct, we need to show that the rewriting algorithm is sound wrt the view definitions. This guarantees that whenever we obtain a solution to the rewritten source-to-target mapping, we can apply the view definitions to obtain an instance of the ontology that is a solution to the source-to-ontology mapping. To prove soundness, we need to prove that the four transformations are sound with respect to the view definitions.

We first notice that *Transformation 1.* corresponds to the standard view unfolding procedure, which is known to be sound.

Transformation 3. and *4.* generate dependencies that are logically equivalent to the original ones. In *Tranformation 3.*, we turn $\phi(\bar{x}) \wedge \neg L(\bar{x}) \rightarrow \bot$ into $\phi(\bar{x}) \rightarrow L(\bar{x})$. Call a the formula $\phi(\bar{x})$, b atom $L(\bar{x})$, then we have that:

$$a \wedge \neg b \rightarrow \bot \equiv \neg(a \wedge \neg b) \equiv \neg a \vee b \equiv a \rightarrow b$$

Similarly, in *Tranformation 4.*, we turn $\phi(\bar{x}) \wedge \neg L(\bar{x}) \rightarrow \bigvee L_i(\bar{x}, \bar{y})$ into $\phi(\bar{x}) \rightarrow \bigvee L_i(\bar{x}, \bar{y}) \vee L(\bar{x})$. Call a the formula $\phi(\bar{x})$, b atom $L(\bar{x})$, and c the formula $\bigvee L_i(\bar{x}, \bar{y})$. Then we have that:

$$a \wedge \neg b \rightarrow c \equiv \neg(a \wedge \neg b) \vee c \equiv \neg a \vee b \vee c \equiv \neg a \vee (b \vee c) \equiv a \rightarrow b \vee c$$

We only need to discuss *Transformation 2.*. This takes a ded of this form:

$$d : \forall x : \phi(\bar{x}) \rightarrow \exists \bar{y} : \bigvee \psi_i(\bar{x}, \bar{y}) \vee (R_0(\bar{x}, \bar{y}) \wedge \ldots \wedge \neg L(\bar{x}, \bar{y}) \wedge \ldots R_k(\bar{x}, \bar{y}))$$

with a negated $\neg L(\bar{x}, \bar{x})$ atom in one of its conjuncts, and replaces it by two dependencies. The first one is obtained from d by replacing $\neg L(\bar{x}, \bar{y})$ by a new atom $TGD_i(\bar{x}, \bar{y})$, where TGD_k is a new relation symbol:

$$d' : \forall x : \phi(\bar{x}) \rightarrow \exists \bar{y} : \bigvee \psi_i(\bar{x}, \bar{y}) \vee (R_0(\bar{x}, \bar{y}) \wedge \ldots \wedge TGD_i(\bar{x}, \bar{y}) \wedge \ldots R_k(\bar{x}, \bar{y}))$$

The second one has the form:

$$d^1 : \forall x, y : L(\bar{x}, \bar{y}), TGD_i(\bar{x}, \bar{y}) \rightarrow \bot$$

It is easy to see that any solution for d', d^1 is also a solution for d. In fact, any solution for d', d^1 must be such that, for any homomorphisms h, facts $h(TGD_i(\bar{x}, \bar{y})), h(L(\bar{x}, \bar{y}))$ are not present at the same time. This implies that either the premise of d is true according to h, and $h(L(\bar{x}, \bar{y}))$ is false, or the opposite. This proves that also *Transformation 2.* is sound.

Since all transformations are sound, algorithm *UnfoldDependencies* is sound and the claim is proven. □

Theorem 3 *Given a source-to-ontology scenario $\mathcal{M}_{SV} = \{\boldsymbol{S}, \boldsymbol{V}, \Sigma_{SV}, \Sigma_V\}$ with view definition Υ_V, assume Υ_V conforms to the restrictions in Definition 4. Call $\mathcal{M}^{emb}_{ST'} = \{\boldsymbol{S}, \boldsymbol{T'}, \Sigma_{ST'}, \Sigma_{T'}\}$, the source-to-target rewritten scenario computed by algorithm UnfoldDependencies, where $\boldsymbol{T'}$ is obtained from $\boldsymbol{T}$ by enriching it with a finite set of new relation symbols $TGD_0, TGD_1, \ldots$. Then $\mathcal{M}^{emb}_{ST'}$ only contains embedded dependencies (i.e. tgds, egds, and denial constraints).*

Proof: Assume Υ_V conforms to Definition 4. We now show that algorithm *UnfoldDependencies* does not introduce any disjunction during the rewriting.

To start, we notice that the original source-to-ontology mapping only contains ordinary embedded dependencies, and therefore no disjunction nor negation is present. Notice also that the premise of source-to-target tgds only contains source symbols, and these are not rewritten.

By looking at algorithm *UnfoldDependencies*, we notice that a disjunction can only be introduced when a dependency $d : \phi(\bar{x}) \rightarrow \exists \bar{y} : \psi(\bar{x}, \bar{y})$ containing a negated atom $\neg L(\bar{x})$ in the premise, and a non-empty conclusion, is rewritten to yield $d' : \phi(\bar{x}) \rightarrow (\exists \bar{y} : \psi(\bar{x}, \bar{y})) \vee L(\bar{x})$.

To see in which cases this may happen, we now want to investigate how the negated atom in the premise of d has appeared in the first place. Recall that the original tgds and egds do not contain negations. By reasoning on the transformations, we notice that this may happen only in two cases:

(i) the first case is the one in which d was originally a denial constraint of the form $d_i : \phi(\bar{x}) \rightarrow \bot$ with two different negated atoms, $L(\bar{x}), L'(\bar{x})$ in the premise; in this case, d_i is initially rewritten to move $L'(\bar{x})$ to the conclusion according to *Tranformation 3.* to yield $d : \phi'(\bar{x}) \rightarrow L'(\bar{x})$, and then also $L(\bar{x})$ according to *Transformation 4.*, to yield d' as discussed above;

(ii) the second case is the one in which d was originally an egd of the form $d_j : \phi(\bar{x}) \rightarrow x = x'$, and $\phi(\bar{x})$ contained a negated atom that is then moved to the conclusion by introducing a disjunction.

Consider first case (i). Recall that denial constraints are introduced exclusively by *Tranformation 2.* when one of the dependencies has a negated atom in its conclusion. Therefore, for case (i) to happen, we need:

- a tgd with a view symbol V in its conclusion, that is unfolded according to *Tranformation 1.* to introduce a negated view atom $\mathsf{V}'(\bar{x}, \bar{y})$;
- atom $\mathsf{V}'(\bar{x}, \bar{y})$ is removed by *Tranformation 2.*, to generate a new tgd d^1 in which it appears positively in the premise;
- atom $\mathsf{V}'(\bar{x}, \bar{y})$ in the premise of d^1 is again unfolded according to *Tranformation 1.*, to introduce two different negated atoms $\neg L(\bar{x}), \neg L'(\bar{x})$ in the premise of d^1;
- these are rewritten according to *Transformation 3.* first, and then *Tranformation 4.*, as discussed above, to generate a ded.

We notice, however, that this is not possible by Definition 4, since it would require a view (V), that negatively depends on another (V'), and this in turn depends on two negated atoms.

Let us now consider case (ii) above. This requires that one of the original egds contains a view symbol that is unfolded to introduce a negated atom in the premise. This is, however, also prevented by Definition 4.

This proves that under the restrictions of Definition 4, no disjunction is introduced by the algorithm, and therefore the resulting set of dependencies is a set of standard embedded dependencies (tgds, egds, and denial constraints). □

B Complete Rewriting for the Running Example

Source schema: *S-WorkerGrades*(*WorkerId*, *Year*, *Grade*, *SalaryInc*)
S-Stats(*WorkerId*, *WorkerName*, *MinGrade*, *MaxGrade*)

Target schema: *Employees*(*Id*, *Name*)
Evaluations(*EmployeeId*, *Year*)
PositiveEvals(*EmployeeId*, *Year*, *SalaryInc*)
Penalized(*EmployeeId*, *Year*)
Warned(*EmployeeId*, *Date*)

View definitions for the target ontology:

$\mathsf{Worker}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathit{Employees}(\mathit{id}, \mathit{name})$
$\mathsf{Evaluation}(\mathsf{workerId}, \mathsf{year}) \Leftarrow \mathit{Evaluations}(\mathit{workerId}, \mathit{year})$
$\mathsf{PositiveEval}(\mathsf{workerId}, \mathsf{year}, \mathsf{salaryInc}) \Leftarrow \mathsf{Evaluation}(\mathsf{workerId}, \mathsf{year}), \mathit{PositiveEvals}(\mathit{workerId}, \mathit{year}, \mathit{salaryInc})$
$\mathsf{NegativeEval}(\mathsf{workerId}, \mathsf{year}) \Leftarrow \mathsf{Evaluation}(\mathsf{workerId}, \mathsf{year}), \neg\mathsf{PositiveEval}(\mathsf{workerId}, \mathsf{year}, \mathsf{salaryInc})$
$\mathsf{Problematic}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \mathit{Penalized}(\mathit{id}, \mathit{year})$
$\mathsf{Problematic}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{PositiveEval}(\mathsf{id}, \mathsf{year}, \mathsf{salaryInc})$
$\mathsf{Outstanding}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{NegativeEval}(\mathsf{id}, \mathsf{year}), \neg \mathit{Warned}(\mathit{id}, \mathit{date})$
$\mathsf{Average}(\mathsf{id}, \mathsf{name}) \Leftarrow \mathsf{Worker}(\mathsf{id}, \mathsf{name}), \neg\mathsf{Outstanding}(\mathsf{id}, \mathsf{name}), \neg\mathsf{Problematic}(\mathsf{id}, \mathsf{name})$
$\mathsf{Year}(\mathsf{number}) \Leftarrow \mathit{Evaluations}(\mathit{employeeId}, \mathit{number})$

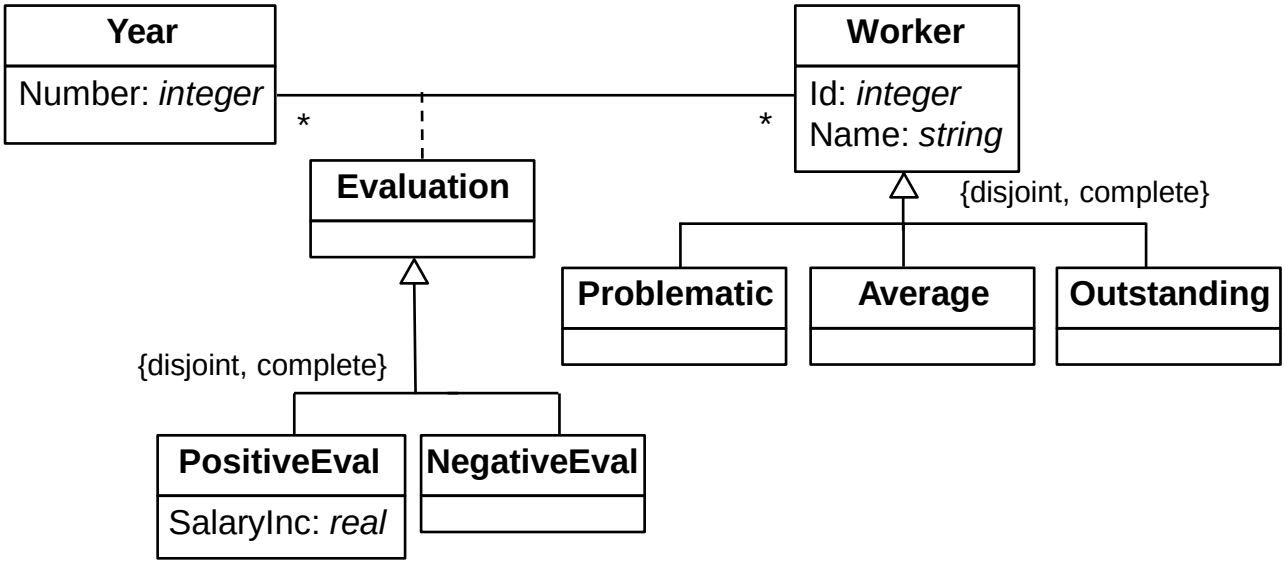


Figure 7: Target Ontology.

Source-to-ontology mapping dependencies:

$$m_0 : \forall id, yr, gr, sinc, name, mingr, maxgr :$$
$$S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr),$$
$$maxgr > 4, mingr < 9 \rightarrow \mathsf{Average(id, name)}, \mathsf{Worker(id, name)}$$
$$m_1 : \forall id, yr, gr, sinc, name, mingr, maxgr :$$
$$S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr),$$
$$mingr \geq 9 \rightarrow \mathsf{Outstanding(id, name)}, \mathsf{Worker(id, name)}$$
$$m_2 : \forall id, yr, gr, sinc, name, mingr, maxgr :$$
$$S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr),$$
$$maxgr \leq 4 \rightarrow \mathsf{Problematic(id, name)}, \mathsf{Worker(id, name)}$$
$$m_3 : \forall id, yr, gr, sinc :$$
$$S\text{-}WorkerGrades(id, yr, gr, sinc), gr \geq 5 \rightarrow \exists name : \mathsf{PositiveEval(id, yr, sinc)},$$
$$\mathsf{Evaluation(id, yr)}, \mathsf{Worker(id, name)}, \mathsf{Year(yr)}$$
$$m_4 : \forall id, yr, gr, sinc :$$
$$S\text{-}WorkerGrades(id, yr, gr, sinc), gr < 5 \rightarrow \exists name : \mathsf{NegativeEval(id, yr)},$$
$$\mathsf{Evaluation(id, yr)}, \mathsf{Worker(id, name)}, \mathsf{Year(yr)}$$

Ontology egds:

$$e_0 : \forall \mathsf{id}, \mathsf{name}_1, \mathsf{name}_2 : \mathsf{Worker(id, name}_1), \mathsf{Worker(id, name}_2) \rightarrow \mathsf{name}_1 = \mathsf{name}_2$$
$$e_1 : \forall \mathsf{id}_1, \mathsf{id}_2, \mathsf{name} : \mathsf{Outstanding(id}_1, \mathsf{name}), \mathsf{Outstanding(id}_2, \mathsf{name}) \rightarrow \mathsf{id}_1 = \mathsf{id}_2$$

Rewriting of the mapping dependencies into source-to-target:

$$
\begin{aligned}
&m_0^1 : \forall id, yr, gr, sinc, name, mingr, maxgr : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr), \\
&\qquad maxgr > 4, mingr < 9 \rightarrow Employees(id, name), TGD_1(id, name) \\
&m_0^2 : \forall id, name : TGD_1(id, name), Employees(id, name) \rightarrow \\
&\qquad \exists yr : (Evaluations(id, yr), TGD_2(id, yr)) \\
&\qquad \vee \exists date : Warned(id, date) \\
&m_0^3 : \forall id, yr : TGD_2(id, yr), Evaluations(id, yr), PositiveEvals(id, yr, sinc) \rightarrow \bot \\
&m_0^4 : \forall id, name, yr : TGD_1(id, name), Employees(id, name), Penalized(id, yr) \rightarrow \bot \\
&m_0^5 : \forall id, name : TGD_1(id, name), Employees(id, name) \rightarrow \\
&\qquad \exists yr, sinc : Evaluations(id, yr), PositiveEvals(id, yr, sinc) \\
&m_1^1 : \forall id, yr, gr, sinc, name, mingr, maxgr : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr), \\
&\qquad mingr \geq 9 \rightarrow Employees(id, name), TGD_3(id) \\
&m_1^2 : \forall id, yr : TGD_3(id), Penalized(id, yr) \rightarrow \bot \\
&m_1^3 : \forall id, yr : TGD_3(id), Evaluations(id, yr) \rightarrow \exists sinc : PositiveEvals(id, yr, sinc) \\
&m_2^1 : \forall id, yr, gr, sinc, name, mingr, maxgr : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), S\text{-}Stats(id, name, mingr, maxgr), \\
&\qquad maxgr \leq 4 \rightarrow \exists yr' : (Employees(id, name), Penalized(id, yr')) \\
&\qquad\qquad \vee (Employees(id, name), TGD_4(id)) \\
&m_2^2 : \forall id, yr : TGD_4(id), Evaluations(id, yr), PositiveEvals(id, yr, sinc) \rightarrow \bot \\
&m_3^1 : \forall id, yr, gr, sinc : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), gr \geq 5 \rightarrow Evaluations(id, yr), \\
&\qquad\qquad PositiveEvals(id, yr, sinc) \\
&m_4^1 : \forall id, yr, gr, sinc : \\
&\quad S\text{-}WorkerGrades(id, yr, gr, sinc), gr < 5 \rightarrow Evaluations(id, yr), TGD_2(id, yr)
\end{aligned}
$$

Rewriting of the ontology egds into target dependencies:

$$
\begin{aligned}
&e_0^1 : \forall id, name_1, name_2 : Employees(id, name_1), Worker(id, name_2) \rightarrow name_1 = name_2 \\
&e_1^1 : \forall id_1, id_2, name : Worker(id_1, name), Worker(id_2, name) \rightarrow id_1 = id_2 \\
&\qquad\qquad \vee \exists year : (Evaluations(id_1, year), TGD_5(id_1, year)) \\
&\qquad\qquad \vee \exists date' : Warned(id_1, date') \\
&\qquad\qquad \vee \exists year : (Evaluations(id_2, year), TGD_5(id_2, year)) \\
&\qquad\qquad \vee \exists date' : Warned(id_2, date') \\
&e_1^2 : \forall id, year : TGD_5(id, year), Evaluations(id, year), \\
&\qquad\qquad PositiveEvals(id, year, sinc) \rightarrow \bot
\end{aligned}
$$

The rewriting of mapping dependencies and ontology egds has been simplified (for readability sake): (1) removed redundant atoms, (2) reused relational symbol TGD_2 in m_4^1 (instead of creating a new TGD_i that would be identical to TGD_2), and similarly, (3) used symbol TGD_5 twice in e_1^1, instead of using TGD_5 and another fresh symbol TGD_6.